SABBATICAL LEAVE REPORT

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In the course of my travel during my sabbatical leave, I visited schools and technical museums as well as talking to students and teachers at every opportunity. In all I visited eleven schools, including one junior high school, one senior high school, three universities, one junior college, and one technical university. I also visited the British Broadcasting Office of Publication in London, England.

The plan laid down in my application for my sabbatical leave called for the main thrust of my study to be in the area of math labs, concentrating on what I might find to help improve our own situation at Mount San Antonio College. Unfortunately, this plan proved somewhat unworkable as almost nowhere in my visits did I find a working math lab. At each opportunity I discussed this with teachers, but I ended up giving information in this area instead of receiving it. So that my time might be of use, I concentrated on learning what I could about other aspects of mathematics with which I am also interested and involved. These were concentrated in the areas of testing, video tape use in teaching, and computer math. I also wished to discover how well the schools were doing in selling their programs to the potential students in their areas.

South-End High School for Boys
South-End-on-Sea, Essex, England
Mr. Drummond, teacher

The schools in South-End-on-Sea are still very traditional, having opted not to form comprehensive schools but instead retaining separate schools for boys and girls as well as separating terminal and college-bound students. The exception was the technical college where some students from each track found themselves. Some (not many) went on to universities.

South-End High School is very much a school designed for the elite students. Its A level courses in mathematics were very much like the courses at Mount San Antonio College, running from calculus to computer science and statistics. The classes for the twelve year old students were ouite like you would find in a freshman high school class here, including basic algebra from well-worn books, some very good students, and some not so good. I spent some time working with one of these groups and felt at home with the material, type of student, and the kind of difficulties they were having. In working up to the fifth form (sixteen and seventeen year olds), I found some classes at work without teachers. The material had been laid out for them, but the teacher was away. The topic was what we would call modern algebra, but they called it pure mathematics. In this area I found the work to be at a level found only in our highest math courses. These students were highly motivated and worked independently very well.

South-End Middle School
Bob Middleton, teacher

This school was for the slower track students. When they left this school, the boys went to work in an apprenticeship program. The boys with whom I talked had very little idea of what they would do when they left school. There appeared to sometimes be quite a long time lapse between the time they left school and the time they were absorbed into the economy. One fellow I talked to had been looking for a job for a year and another had been delivering telegrams on a motorcycle. There appeared to be many areas of help for them, but for the most part they did not take advantage of them. This was in marked contrast to the boys at South-End High School across town.

Of particular interest to me in one of the classrooms I visited was the fact that the chalkboards were replaced by special white plastic sheets which you wrote on with special felt pens. The erasing was done with a cloth, thus avoiding the dust of a regular chalkboard. The principal disadvantage appeared to be the odor of the ink. I brought back a sample of this to try in my classroom.

There was some mobility in the system as a few of these 'slower' students went to the technical college, and it was possible to go from there to the university but few apparently did. The mathematics taught to these boys appeared to be traditional math with some algebraic techniques being used. In the last year some geometry and numerical trigonometry were taught. There were no computers available to the students at this school.

Mr. Middleton used video tapes in teaching arithmetic, and I was directed to the British Broadcasting office to preview the tapes. Following is a summary of that preview session.

Video Tape Preview
B. B. C. Publications, London

The video tape is designed for ages 12 to 15; the topic is mathematics. The tape was of higher quality than any other I have ever seen. The actors were young, professional actors. The script was about a young couple that the students could easily identify with, both in age and vocabulary. The script had a plot that would capture the attention of the students, and the math was presented throughout as being used by a young apprentice carpenter. The accent of the actors would probably prevent using these films with American students. It was obvious that many hundreds of dollars were spent on each film.

Tape Topics and Notes

Tape 1: Pounds and Pence

Topics: currency notation, multiplication of a whole number by a whole number, division of whole numbers, multiplication of decimals.

Sam is calculating with a pencil and paper. He says one pound five pence and writes 1.5, then multiplies: $1.5 \times 6 = 9.0$. Mike disagrees. He visualizes a L:1 note and a 5 p coin multiplied by 6 to give six L:1 notes and six 5 p coins or 6 pounds 30 pence. Note the negative example followed by the correct example. After some hand calculation the work then turned to the calculator.

Tape 2: The Long and Short of It

Topics: measuring in metric then converting to feet.

The same boys that were in the first tape are moving out on their own. Mike goes to work in a park and Sam does carpentry work. The tape starts with Sam working out the cost of living. I got the distinct feeling that any student contemplating going out on his own would have been interested in the tape. The boys were very real English boys, and they, as well as the English girls, were attractive and appealing. This is certainly a far cry from what we find in our tapes or in tapes we rent from commercial sources.

These tapes and others are of clearly superior quality but could not be used directly, as the topics and language would not be suitable for our students. If copies of scripts could be obtained, they could be altered and produced with language more in line with an American student audience. This would be expensive, as neither students nor teachers could be used in the tape or the quality would be bound to suffer. A trial tape could be developed with the drama department students. This might be satisfactory.

German University
Seigen, West Germany
Dr. Ostholthoff

I spent two days with Dr. Ostholthoff. I visited the computer center, library, hydraulics lab, and the mathematics and chemistry departments. Classes were not in session, but I spent many hours with various people at the university. Of particular interest was the computer center and the hydraulics lab. The computer was a central system of German make with terminals in the library and hydraulics lab but none in the math area. Dr. Ostholthoff said he believed there were fifteen to eighteen terminals around the campus. The terminal in the library appeared to be under very light control, but in the regular session perhaps the situation would be different. The computer was used principally by engineering students and computer science students. The math area did not incorporate it into their classes to any degree. Also of interest was the lack of computer work in the economic area. Graduate degrees were offered but with no identifiable computer or mathematics background. The hydraulics lab was very interesting, with a scale model of a dam and hydroelectric system. I was told that a complete computer model of the system was on line in their terminal.

The university was generally very plain architecturally. The library had large exposed air conditioning ducts, and the grounds were grown-up with weeds. The site, one of the best I have ever seen, was situated on a hill overlooking the city of Seigen, an old city that was destroyed by allied bombs and completely rebuilt. Dr. Ostholthoff was very appreciative of the help given by the Americans in rebuilding the city.

I am expecting a visit from him in the future and look forward to showing him Mount San Antonio College. He and his wife, Toni, were indeed gracious hosts.

Conclusion:

The widespread placement of terminals did not mean that they would be effectively used. The university's program in economics could be improved by the inclusion of courses like finite math, analysis, and statistics.

Nurenburg College
Nurenberg, West Germany
Dr. Heinz Rausch

The college was situated within the old walled city, and although it was damaged in the war, it was rebuilt to blend with the pleasant old city. I spent the evening visiting with Dr. Rausch at his home and was amazed at the similarity between his teaching, life style, and living accommodations and ours here. He taught twelve hours of class and was required to hold two office hours per week. No sabbatical was available; however, he and his wife traveled extensively throughout Europe during vacations. The college offered some lower level math much like our Math 51 and 52. The geometry was less rigorous and more numerical. There appeared to be many theorems taught that were of little practical or developmental value. The computer center was closed for repair. The school seemed to specialize in social science (called social medicine), but there was an engineering school in another part of the city. The students were not required to take statistics or any background work in computers. Some of the classes had units in various mathematic topics. Classes were in German.

Patrai University
Patrai, Greece

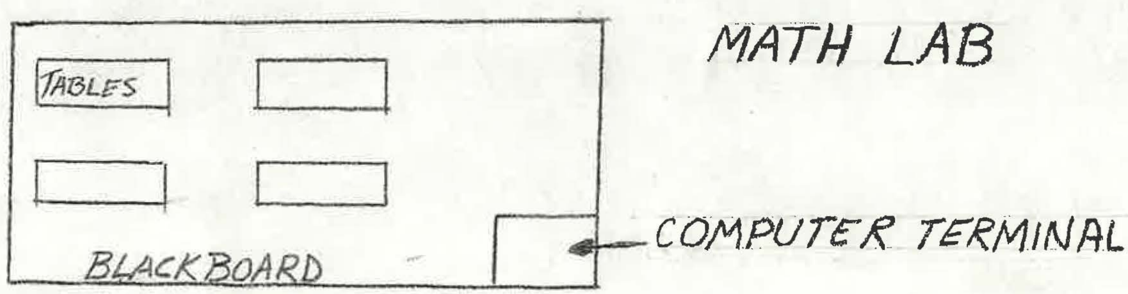
I visited with the computer operator, who knew very little English. The computer terminal was in the library and had students crowded around it. Greek universities have a system that requires a student to pass all classes or repeat the entire year. This puts the students under heavy pressure and results in picketing and strikes. Passing is dependent on a series test at the end of the year. I watched an engineering class (thermodynamics) conducted in Greek but could not get too much out of it. The students were unruly and noisy.

Technical University
Jerusalem, Israel

This school is situated on a hillside outside Jerusalem, within sight of the Knesset building. It is without a doubt the most beautiful university that I visited. The school was principally built with funds from the United States, both public and private.

This was the one place that I encountered a type of math lab. The area was informal and included an area for students to congregate and work on math and science problems. There was no trained personnel, but the students helped each other. There was a teacher stationed nearby that could give help when it was required. At the time I was there, a student was in the process of working a partial-differential equation on the blackboard with the coaching of four other students. There was also a computer installation nearby (IBM 370).

Although the students at this university were not of the same type found in the math lab at Mount San Antonio College, it is clear that many of our students could be helped by this kind of a system.



It would require no staff but would require some area set aside. It would be useful for it to be located near the math lab and offices, as well as near some kind of computer facility. The idea is for the students not to get help from the lab instructors, as I believe this would prove to be a crutch that would in the long run prove detrimental to the students by keeping them from developing good problem solving techniques. The computer terminal is also something that could be omitted, but students should have the computer available to develop the heuristic techniques using computer methods.

Upon our arrival at Heathrow Airport, outside London, England, we set out to meet our first objective of the trip, the purchase of a camping vehicle. Our family abroad consisted of myself, my wife Marilyn, and three of our four children, Melinda age sixteen, Debra age thirteen, and Mark age 6. We needed a van large enough for five people to travel, live, and work in for seven months, with at least one bed large enough to accommodate a six footer, and a left wheel drive. After several days of non-stop looking in the English damp and drizzle, we finally found a vehicle that was nearly perfect, the two flaws being that it was a right wheel drive, and, in the words of Melinda, that it looked like an ice cream truck. So, off we went, bag, baggage, and guitar case, down the road with driver and van on the "wrong" side.

Our itinerary, always subject to instant change, took us along the cliffs and beaches of England's southern shores, onto the Salisbury Plain to Winchester and Stonebenge, thru the hedgegrows ripe with wildflowers of Cornwall, to the bleak Bodmin Moor. We wound our way along the Welsch coastline then darted north to the Lake District, always marveling at the never ending green carpet covering the hillsides, the endless fields of scarlet iceland poppies bordered by countless varieties of wildflowers along the roadside and the great abundance of water, both falling from the sky and in the form of lakes, rivers, and streams. We are native southern Californians and are not accustomed to the luxuriant side of Mother Nature.

A circuit through Scotland included the glens and lochs of the rugged western coast and the rolling heather-covered hills of the interior, then down the Eastern side of the island, through Edinburgh and across the moors and marshes, stopping at Whitley and York to recapture the past. Then we proceeded to London.

By now we had established a routine and pattern which included visiting schools, school work for the children, touring, visiting with new friends and students in the campgrounds, and making side trips to swim teams and square dances, as our children are competitive swimmers, and my wife and I are square dancers. These last two activities not only provided recreation but were an important way of meeting people, both socially and professionally. We continued to follow this pattern throughout our trip.

Prior to crossing the channel, we spent several days with Penny Dean, a former swim coach of our eldest daughter. Penny was preparing to swim the English Channel and had been in training at Folkstone for four months. After the swim had been canceled several times because of adverse weather conditions, we finally crossed, only to discover the following day that Penny had followed us and in spite of a chronic cold and cough and troublesome shoulders, had broken all records for a single crossing by over an hour! Truly an enviable achievement.

Our first destination on the continent was a small town in Holland to visit a family we had met on a previous European trip. We spent a week there, enjoying the family and a local festival. The family owned and operated a large campground which housed a large teen-age population, which, needless to say, delighted our teen-age daughters and gave me time to catch up on my work. We were invited to participate in activities planned for a sister city delegation from Siegen, Germany and thus became acquainted with a university professor and his wife, both of whom we greatly enjoyed. Later in our trip we spent several days at the Ostholthoff's home in Siegen. Our Dutch hostess, a member of the local city council, told us that at first there had been considerable resistance in the town to the idea of a German sister city but that now the program was working well.

After leaving Holland, we spent time in France, Belgium, Germany, and Denmark. One of the highlights of this part of our trip was a brief stay in East Germany, which included Wittenburg, the residence of Martin Luther for many years. Here we had a chance to explore some of our religious "roots" and heritage. Although the communist society is atheistic, it recognizes the tourist potential of this center of the Protestant Reformation, and the sites are reasonably well maintained by western standards. The Russian military was highly visible throughout the portion of Eastern Germany that we traveled through. We later learned they were having summer maneuvers, as were our allies in West Germany.

We spent a week in West Berlin, a great favorite with the entire family. It is a city of growth and vitality, fully aware of its precarious position. Besides touring and shopping we square danced and attended the World Championship

Swim Meet, where the young American swimmers swept the meet.

Next came Switzerland, Austria, Hungary, and Yugoslavia, where we hurriedly dashed south to escape the ever present rain. Laundromats were becoming increasingly few and far between, and we were all feeling slightly moldy and out-of-sorts because of the continual grey skies, wet clothes, and mud. In Greece we had a great deal of R and R, along with some touring and shopping. While they shopped, I did school work. While in Athens, we decided to go to Israel and made arrangements to transport ourselves and our van on a ferry, with a brief stopover on Rhodes.

Israel was, without a doubt, one of the most interesting parts of the trip, so much variety, both in people and topography, packed into such a small country. We camped throughout the country and everywhere the people were open and hospitable. There seemed to be great curiosity about ourselves and the van. On one occasion an Israeli wanted to buy our refrigerator. While camped on the Sea of Gallilee, we shared a fish barbecue with a group of Israelis who danced around the campfire and sang peace songs in Hebrew.

All too soon it was time to head back north and by Christmas we were back in London. Shortly after the new year began, we sold the van to an Australian and headed home.

We have pleasant memories of the countries visited, sites seen for the first time, and favorite places re-visited, but most of all we remember the people we met in the campgrounds, the strangers who shared their food, the new friends who opened their homes to us, and my overseas colleagues who enriched my knowledge.

Ways This Sabbatical Relates to My Teaching at Mt. San Antonio College

This trip should prove valuable to me in my teaching assignment at Mt. San Antonio College in several ways. First, and perhaps most important, the ongoing contact with teachers on the continent should provide me with fresh ideas for years to come. In mathematics fresh ideas are not as readily available as in many disciplines, and we tend to settle into a pattern that may not be the best one for our students. The curriculum materials, tests, and outlines that I obtained on this trip should lay a solid foundation for many improvements in my teaching and serve as a spring board for curriculum changes in the mathematics department.

The second major area was in addition to ideas for teaching math on video tape. The mathematics lab at Mt. Sac is already moving in a direction that should make it much more effective. The lab is depending on the video tapes to teach mathematics and in order to do this, fresh ideas are necessary. I believe the video tapes I viewed at the BBC can serve as a model to make this learning more effective. We must move the teacher off the center stage on the screen and move the students onto it, as the British have done.

Another area where things that I learned can be of use is in improving the learning atmosphere for the better students in the mathematics area. The model that I found in Jerusalem could lead to this improvement. (See Page 8) An area needs to be set aside where the students can go work together. Space is hard to come by, but I will continue to work on this objective. This should help in both recruitment and retention of the student group we are now missing in mathematics, the superior student.

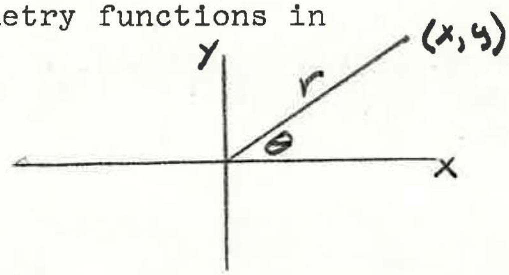
A fact I learned while talking to young people away from schools could also help us at Mt. Sac. I discovered that many programs existed in schools that many, if not most, people did not know existed. I found myself telling them what was available in their own schools. We should investigate this problem in our district, and if it exists, as it surely does, we should move to correct this situation by doing a better job of informing the community about our special programs.

Lastly, I might say that the enthusiasm I have gained from my experience should serve to improve my teaching. I was lifted out of a rut I did not even know I was in, and the feeling was terrific!

Review Tape Trigonometry

The purpose of this tape is to review the concepts of trigonometry in order to prepare you for the calculus. It is not designed to teach you trigonometry. We have courses and a whole series of tapes for that purpose.

First, the definitions of the trigonometry functions in terms of the x-y co-ordinate plane.



$\sin\theta = y/r \qquad \csc\theta = r/y$

$\cos\theta = x/r \qquad \sec\theta = r/x$

$\tan\theta = y/x \qquad \cot\theta = x/y$

These definitions allow you to solve many problems:

Example: given $\sin\theta = +3/4$ in QUAD II

Solution: the third side of the triangle is found using the theorem of Pythagores

$$x^2 + 3^2 = 4^2$$
$$x^2 + 9 = 16$$
$$x^2 = 17$$
$$x = \pm\sqrt{17}$$

But since θ in QUAD II $x = -\sqrt{17}$ thus

$\cos\theta = -\sqrt{17}/4$, $\tan\theta = \dfrac{3}{-\sqrt{17}}$, $\csc\theta = 4/3$,

$\sec\theta = ?$, $\cot\theta = ?$

These definitions lead directly to the following identities. You should know these:

$$\sin^2\theta + \cos^2\theta = 1; \quad 1 + \tan^2\theta = \sec^2\theta \quad \cot^2\theta + 1 = \csc^2\theta$$ Pythagoras identities

$$\sin\theta = 1/\csc\theta \quad \cos\theta = 1/\sec\theta \quad \tan\theta = 1/\cot\theta$$ Recipocal identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$ Quotient identities

Example of proof:

$x^2 + y^2 = r^2$	thereom of Pythagoras
$1 + \frac{y^2}{x^2} = \frac{r^2}{x^2}$	dividing each side by x^2
$1 + \tan^2\theta = \sec^2\theta$	definition of tanθ and secθ

Directly from the definition we obtain the following table:

	sin	cos	tan	sec	csc	cot
0	0	1	0	1	---	---
30						
45						
60						
90						
120						
135						
150						
180						
210						
225						
240						
270						
300						
315						
330						

Example: $\sin(210) = -\frac{1}{2}$

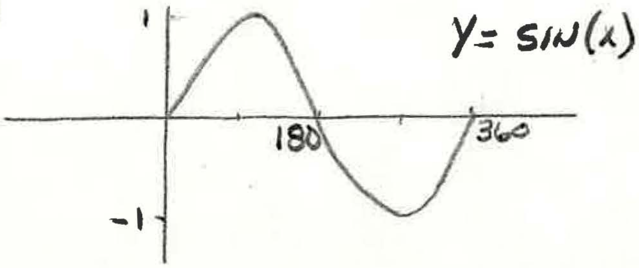
Remember in 30-60-90 $\triangle$ sides are in the Ratio 1-1- 2
2 $\sqrt{3}$

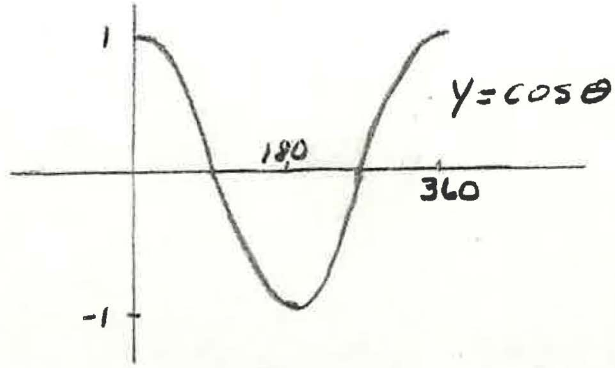
Example: $\tan(270) = y/x = -1/0$

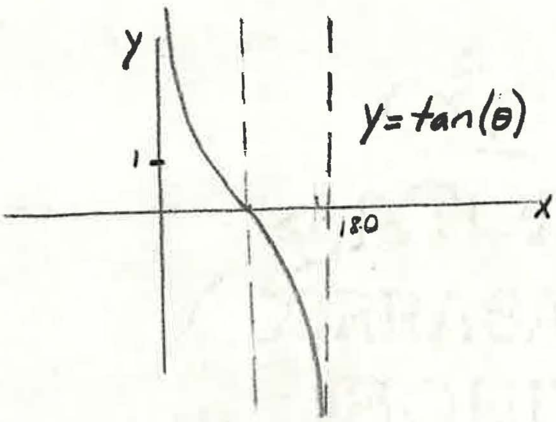
$x = 0,\ y = -1,\ r = 1$ not defined

You should <u>know</u> how to fill out this table before you take the calculus test or start calculus.

Graphing Trigonometry Functions







You should graph
y=cot θ, y=sec θ, y=csc θ

These are sketches of the basic trigonometry functions. Study them. All of the data for graphing them is in Table I.

Note the following:

y = sinx   Period 2π (The period is the x distance for the functions to make one complete pattern y direction)

Amplitude   1. This is the maximum displacement from the axis of symmetry.

y = cosx   period 2π, amp 1

y = tanx   period π, amp not defined

y = cotx   period π, amp not defined

y = secx   period 2π, amp not defined

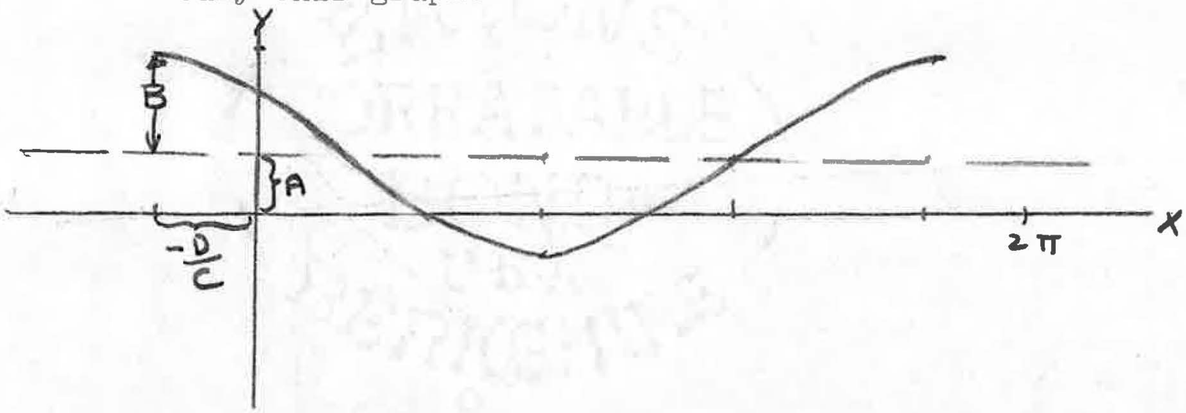
y = cscx   period 2π, amp not defined

Example: graph $y = A + B \cos (Cx + D)$

Note:
- A shifts curve up
- B changes amplitude to B
- C change period to $\frac{2\pi}{C}$ (divide normal period by C)
- $C;D$ displace curve $-\frac{D}{C}$ to right or left

(This is where $cx + 1) = 0$

Study this graph.



The following formulas are also necessary:

sum formulas

$$\sin (A+B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A+B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

difference formulas

$$\sin (A-B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A-B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

double angle formulas

$$\sin (2A) = 2 \sin A \cos A$$
$$\cos (2A) = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A = \cos^2 A - \sin^2 A$$
$$\tan (2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin (\tfrac{1}{2}A) = \pm \frac{1 - \cos A}{2}$$

$$\cos (\tfrac{1}{2}A) = \pm \frac{1 - \cos A}{2}$$

$$\tan (\tfrac{1}{2}A) = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Additional Topics

Radian Measure. Another unit for measuring angles is the Radian:

One Radian is the angle where the arc of a circle intersects an arc equal to the radius.

This results in π Radians = 180^0

Example: find 39^0 in Radius.

Using ratio $\frac{\pi}{180} = \frac{x}{39}$ $\quad x = \frac{39}{180}\pi$ Rad.

Example: find 2.31 Rad. in degrees:

$\frac{\pi}{180} = \frac{2.31}{x}$ $\quad \underline{\underline{x = 132.3^0}}$

Trigometric Equations

Definition Identity: an equation that is true for all values of the variable where it is defined:

Example: $\sin^2\theta + \cos^2\theta = 1$

Definition Conditional Equation: an equation that is not an identity.

Example: $\cos 2\theta + \cos\theta = 1$

Solution: using identity $\cos 2\theta = 2\cos^2\theta - 1$

yields $2\cos^2\theta - 1 + \cos\theta = 1$

$2\cos^2\theta + \cos\theta - 2 = 0$

using quadratic formula with a=2 b=1 c=-2

yields $\cos\theta = \frac{-1 \pm \sqrt{1^2 - 4.2.(-2)}}{2.2}$

$= \frac{-1 \pm \sqrt{17}}{4}$

$\cos\theta = .7808, -1.28$

38.7^0 Quad 1
321.3^0 Quad 4

Note: $\cos\theta = -1.28$

Does not yield a solution since -1 cos 1

Examples: Prove identity

Prove: $\sin\theta = 2\tan\frac{1}{2}\theta/(1 + \tan^2\frac{1}{2}\theta)$

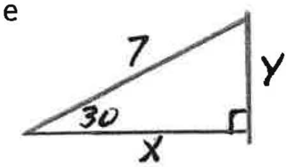
$$= \frac{2\,\dfrac{\sin(\frac{1}{2}\theta)}{\cos(\frac{1}{2}\theta)}}{1 + \dfrac{\sin^2\frac{1}{2}\theta}{\cos^2\frac{1}{2}\theta}} = 2\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta$$

$= \sin\theta$ using #1 above

Solving triangles:

Right triangles can be solved by use of the definitions of the trigonometry functions.

Example



$\sin 30 = \frac{y}{7}$ $\quad y = 7\sin(30) = 7\,(\frac{1}{2}) = 3.5$

$\cos 30 = \frac{x}{7}$ $\quad y = 7\cos(30) = 7\cdot\frac{\sqrt{3}}{2}$

Oblique triangles:

The formulas:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ Law of Sines

$$a^2 = b^2 + c^2 - 2bc\cos A$$ Law of Cosines

May be used to solve triangles that are not right triangles.

Example given triangle $LA = 35^0$, $LB = 40^0$ $a = 15$

find b.

using law of Sines: $\frac{15}{\sin 35} = \frac{b}{\sin 40}$ yeilds $b = 16.8$

Example II Given $a = 17$ $b = 14$ $LC = 32^0$

find C

Law of cosines:

$c^2 = a^2 + b^2 - 2\,ab\cos C$

$c^2 = 17^2 + 14^2 - 2\,.\,17\,.\,14\cos 32^0$

$c = 9.02$ (using calculator or tables for $\cos 32^0$)

Care must be taken when you are given two sides of a trangle and the angle opposite one of them. If the side opposite the given angle is less than the side adjacent to the given angle, two solutions may result. Check a trigonometry book or review the appropriate tapes to review this concept.