REPORT OF SABBATICAL LEAVE

John W. Burns

Fall, 1987

"I Touch the Future;
I am a teacher."

Krystal McAuliffe

Killed-in-Action
Aboard the Space Shuttle, Challenger
January 27, 1986.

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Introduction

I want to thank the College, the Administration, the Board of Trustees, and the Sabbatical Leave Committee for granting me the opportunity to return to school for full-time studies to broaden my knowledge, modernize my views, and improve my teaching techniques. I am fully cognizant this was both an honor and a priviledge.

I utilized this leave to the greatest possible extent, both in the quantity and the quality of courses that I took.

I chose UCLA for my studies because I wanted the best possible education that I could obtain. Evidently, I chose well.

I was pleased to learn that according to the 1986 ratings, UCLA rose to the rank of fourth best Graduate School, and the tenth best Undergraduate school in the United States. (Although the thought is flattering, it is unlikely that my matriculation influenced either rating.)

According to the terms of the Sabbitical Leave Agreement, I was obligated to take twelve semester units each semester, for a total of twenty-four semester units (thirty-six quarter units). However, since I wanted to derive the maximum possible benefit from my Leave, I took twenty quarter units each quarter for a total of sixty quarter units, (forty semester units). This was approximately one and two-thirds times the required amount. Among the fifteen courses that I took, eleven were at the Graduate level.

In my selection of courses, I was doubly fortunate. First, in these fifteen courses, I received ten A's, and 2 B's. (I haven't received grades for the other three classes.) More important, I had fourteen superb teachers. (In my rating scale, "Superb" is one step above excellent.)

Shortly before I enrolled at UCLA, it was decided that since the writing ability of college graduates was suspect, professors should be encouraged to assign term papers. The Faculty snapped up this suggestion. In the fifteen courses I took, I wrote thirty term papers. The shortest of these was three pages, and the longest was fifty-three pages. Since these papers played such a large role in the learning process, several are included in this report to indicate the nature and depth of my studies, and to show some of the results.

MT. SAN ANTONIO COLLEGE
Salary and Leaves Committee

MT. SAN ANTONIO COLLEGE
[illegible] -2 PM 3:25
PERSONNEL OFFICE

APPLICATION FOR SABBATICAL LEAVE

Name of Applicant John W. Burns

Address 614 N Charter Drive, Covina, California 91724

Employed at Mt. San Antonio College beginning 1973

Dates of last sabbatical leave: None

From ______ To ______

Department Mathematics-Astronomy

Division Natural Sciences Division

Length of sabbatical leave requested:

One semester ______
Fall ______ Spring ______

Two semesters X

Administrative ______

Purpose of sabbatical leave:

Study X

Independent Study and Research ______

Travel ______

Combination (specify) ______

NOTE: Sabbatical periods are limited to contractual dates of the academic year.

Effective dates for proposed sabbatical leave:

From ______ To ______

and (if taken over a two school year period)

From September, 1986 To June, 1986

Attach a comprehensive, written statement of the proposed sabbatical activity(ies) including a description of the nature of the activity(ies), a timeline of the activity(ies), an itinerary, if applicable, the proposed research design and method(s) of investigation, if applicable.

Attach a statement of the anticipated value and benefit of the proposed sabbatical activity(ies) to the applicant, his/her department or service area, and the College.

Any change or modification of the proposed sabbatical activity(ies) as evaluated and approved by the Salary and Leaves Committee must be submitted to the Committee for reconsideration.

John W Burns

Signature of Applicant John W. Burns

Date December 2, 1985

Applicant's Name John Burns

The acknowledgment signatures reflect awareness of the sabbatical plan for the purpose of personnel replacement. Comments requested allow for recommendations pertaining to the value of the sabbatical leave plan to the College. Applicants must obtain the signatures of acknowledgment prior to submitting application to the Salary and Leaves Committee.

ACKNOWLEDGMENT BY THE DEPARTMENT/DIVISION

Signature of Department Chairperson Larry Heyman Date 12/20/85

Comments:

Signature of Division Dean Barbara Crane Date 12/20/85

Comments: letter to follow

ACKNOWLEDGMENT BY THE OFFICE OF INSTRUCTION

Signature of Asst. Superintendent/Vice President, Instructional & Student Services J A Majorski Date 12-20-85

Comments:

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FINAL ACTION BY THE SALARY AND LEAVES COMMITTEE:

x Recommend approval to the Board of Trustees

___ Not recommend approval to the Board of Trustees

Walter K. Coleman 2/86
Signature - Chairperson, Salary and Leaves Committee Date

John D Randall 2/86
Signature - Authorized Agent for the Board Date

myw
10/16/85

PROPOSED SABBATICAL LEAVE ACTIVITIES
for
John W. Burns

I am requesting Sabbatical Leave for the 1986-87 academic year in order to undertake full-time studies at UCLA.

1. For the last ten years, I have been teaching both Mathematics and Astronomy here at MSAC. Consequently, I propose to take the majority of my course work in these two fields. Please observe that the Math list greatly outnumbers the Astronomy list.

2. However, I wish to broaden and support this program by taking related courses in Earth, Planetary and Space Sciences, as well as the History of Astronomy and Mathematics.

3. Furthermore, I hope to extend and enrich my general teaching ability by taking courses in Education in general, and "Methods" in particular.

Courses to be Taken:

I plan to take some, but not all, of my courses from the following list. Although all these courses should be offered next year, and exact list of days and times is not yet available. Hence, scheduling conflicts may occur. Consequently, I will make my actual course selection from this list based on compatibility, balance, and interactions.

Note 1: Each of these classes meets three hours per week for one quarter.

Note 2: Lower Division courses have numbers less than 100.
Upper Division courses have numbers between 100 and 199.
Graduate courses have numbers 200 and above.

Astronomy

4 H Topics in Contemporary Astrophysics

An Honors course giving in depth treatment of elementary topics.
(I want to take this course to see how experts present their topics to beginners.)

101 General Astronomy and Astrophysics

A survey of the whole field of astronomy

Earth and Space Sciences

3 Evolution of the Solar System, the Earth, and Life

For the general University student
(I want to take this course to see how Astronomy is interfaced with Geology and Biology.)

Mathematics

33 Infinite Series

Infinite sequences and series

100 The Nature of Mathematics

A teacher training course designed to acquaint students with the nature of modern mathematics and the mathematical method

101 Topics in Algebra

Intended for prospective teachers group theory, number systems, relations, elementary number theory, etc.

106 History of Mathematics

Topics in the history of mathematics and the development of modern mathematics

142 Introduction to Applied Mathematics

An introduction to the fundamental principles and the spirit of applied mathematics.

370 The teaching of Mathematics

A professional course in methods.

Note: While much of the information in these Mathematics courses will be new to me, my primary purpose in taking them will be for the insight they will give me in teaching theory, philosophy and especially methods.

Courses to be Taken: (Continued)

Education

113 Psychological Foundations of Education

Analysis of learning processes in school situations

121 Learning and Education

Models of learning, reinforcement, motifation, individual differences and insruction, etc.

Geophysics and Planetary Sciences

101 Introduction to Geophysics and Space Physics

A survey of geophysics, the physics of the planets, their atmospheres, and the interplanetary medium, with emphasis on topics of current interest.

131 Geochemistry

Origin and abundance of the elements and their isotopes, distribution and reactions of the elements in the earth, oceans, and atmosphere.

History

3 Introduction to the History of Science

A study of the beginnings of the physical sciences involving the transformation from Aristotelian to Newtonian cosmology.

General Rationale for Course Selection

I am teaching in both the fields of math and astronomy, therefore, I propose to take course work in both these fields--as well as courses in the closely related fields of Earth Sciences and Space Sciences. These latter two fields will greatly extend my knowledge and increase my appreciation of astronomical topics. Moreover, since these courses will be mathematical in nature, they will show me additional applications of mathematical methods and techniques. Some of these I hope to be able to utilize in my math courses.

I wish to take History of Science and History of Math because I expect to use information which I obtain from them in my courses. It has been said that one cannot really understand a subject until he knows something of its history. This, I wish to do.

I have taken many courses in astronomy. However, these have been taken as topics of interest to me. In certain areas, I have a specialized knowledge of that topic. However, I do not have a uniform knowledge throughout the entire field. I propose to take a few broad general courses in astronomy to make certain that any gaps that may exist in my knowledge of the field will be identified, and then remedied.

I have chosen these math courses for the following reasons:

1. Since its been 25 years since I obtained my degree, I feel an urge to take additional course work to increase my knowledge in both breadth and depth.

2. I wish to see how emphasis in mathematics has changed.

3. I want to be exposed to a variety of teaching philosophies and techniques.

Rationale for Including Lower Division Courses

1. Astronomy 4H — Honors course in Astronomy and Astrophysics.

   This course will cover many of the difficult topics which I cover in teaching my course in astronomy here at MSAC. I will be able to observe first hand how others organize these topics, explain them, and limit the depth of coverage to levels consistent with the interests and abilities of beginning students. Also, this course is a defacto prerequisite for Astronomy 101, General Astronomy and Astrophysics, which I plan on taking.

2. Earth and Space Sciences 3 — Evolution of the Solar System.

   Much of what I said above will also apply here. However, since this course will be taught by people with Geology backgrounds, I expect to obtain many new insights into these problems. Hopefully, many of these will be useful to better explain, or give different viewpoints to my students.

3. History 3 -- History of Science

I really want to take History of Science to study abbut the origins of Greek studies in astronomy and how they formed the foundation for all further developments in the field. (I already spend a week discussing these ideas in Astro 5. I'm sure I could do a smoother job of it after completing this course. No other course taught at UCLA discusses these topics.

4. Math 33 -- Infinite Series

This is a course which I have wanted to take for years. I have an interest in infinite series, and I would like to study them further in order to understand their many subtleties. Moreover, this course would be useful to me in teaching our Math 33C and Math 34.

Benefit to Myself

In these courses, I hope to acquire five results:

A. Astronomy

1. Definition of current status of the baseline in astronomical studies. What am I leaving out that I should be including, and what am I including that I should be leaving out in my courses.

2. Determination of, and removal of, any gaps that may exist in my knowledge of rudiments of the field due to my "cafeteria style" studies in the field. I do not have a degree in astronomy. I began teaching astronomy when asked to do so by the Chairman of the Math Department here at MSAC.

3. Factual updating—No field of learning is producing new data as rapidly as astronomy.

4. Revision of theory--How has this flood of new facts required modification of theories formulated previously to their discovery?

5. Teaching techniques--How do some people who know more astronomy than I do, and who have been teaching it longer than I have, react to common classroom problems? What teaching methods have they developed that would be beneficial to my students?

Benefit to Myself (Continued)

B. Mathematics

I have taken only three math courses in the last twenty-five years. Interests vary and styles change over such a time interval. All of the above considerations (except 4) would also apply to math. Most of my courses in math were advanced courses meant to cover the broad fields of mathematics.

I took them because I wanted to understand the overall structure of mathematics, and to explore its more productive topics. These courses all approached math from the viewpoint of an advanced scholar who wished to specialize. None of this was aimed at teaching mathematics.

Most of the Math courses on my list are courses that are designed for the teaching--not just the understanding--of mathematics. I have never taken courses of this nature before. I and my students probably would benefit much from the added insight these courses would furnish to me.

C. All Fields

All of the courses on my list should contribute additional information that I can have use in my courses in mathematics, or astronomy, or both! Moreover, taking them should also increase my organization, systemization, and correlation of this information. I expect to obtain many different ideas of teaching philosophy, as well as teaching methods and techniques. All of these should enable me to fulfill my duties better.

Benefit to the Department

A. Course Emphasis

While mathematics changes ever so slightly in 25 years, trends in math do change. Opinions change as to what topics in math are essential, useful, and obsolete for people who will be solving research and engineering problems well into the 21st Century. What better place to find out how math is interacting, with other disciplines, than at a major research university such as UCLA where the faculty consult with other universities, government, or industry on a continuing basis. Such a faculty is aware of what topics in math are becoming more important, and what topics seem to have decreasing importance in the future. Naturally, college courses and college curricula should change to reflect these changes.

I would avail myself of this knowledge during my year at UCLA, and I would share it with fellow faculty members.

Not only would this guarantee that our math education was up-to-date, but it would also expedite our students' transfer to--and success in--four-year universities, as well as success on the job.

To the best of my knowledge, only one member of our Department has taken a Sabbatical to study mathematics during the last ten years, and he is now retired.

Benefit to the Department (Continued)

B. Curriculum Modification

As our society and our technology evolve--as is happening rapidly today--many things change. Not only do a variety of new problems emerge, but interest in certain old problems declines. It is very hard for a college teacher to find out that he is teaching his students how to solve problems that industry no longer wants answered. No only must he introduce new material but he must also prune out that which has become unimportant. A University, such as UCLA, which has a large highly respected Graduate School, and whose Faculty interact considerably with industry and government is the logical place to reappraise student needs and goals in light of current trends.

I feel that such knowledge could be of considerable importance to my Department in instituting new courses, as well as modifying those presently offered.

Benefit to the College

Few things give a school a worse reputation than teaching errors. I have said repeatedly that no field is changing as fast as astronomy. For example, when I began teaching at MSAC in 1973, Saturn had nine moons. When I first taught Astronomy in 1975, it had ten moons. When Gil became Dean of Instruction, and Barbara became Dean of Natural Sciences, Saturn had 17 moons. Today, it has 21!

Can you imagine the impression the Faculty at four-year colleges would have of MSAC if our transfer students tell them that their teacher told them Saturn has 12 moons?

I think that in order to fulfill our obligations to our students, and to maintain our pressent high esteem, we should have a faculty member in each Department aware of current progress in that field, as well as its implication for future modifications and trends in the curricula.

This is not to imply that our Faculty does not keep up with progress. Far from it! However, insight into **long range** modifications, trends, and inter disciplinary relationships sometimes requires full-time effort.

I am both willing and able to serve that role in Mathematics and Astronomy.

Thank you for your consideration.

John W. Burns

John W. Burns

ADDENDUM TO SABBATICAL LEAVE APPLICATION
for
John W. Burns

As indicated in your letter, I wish to expand my Sabbatical Leave Application:

1. Done

2. Done

3. I am requesting a full year Sabbatical leave. I am aware that this requires my taking 2 x 12 hours = 24 total semester hours. (This is the equivalent of 36 total quarter units based on the UCLA system.)

4. I would share any further insights I acquire at UCLA into trends in teaching with my colleagues in any way they prefer: individually, or collectively; orally, or written.

   I would guess that the most effective way would be by means of a talk at one of our Department meetings, followed by individual discussions as desired.

5. I would like to augment my prospective course list by the following additions:

ASTRONOMY

3 The Universe of Stars and Stellar Systems
   A more advanced treatment of stars and stellar systems. (This is a sequel to Astronomy 4H which is on my original list.

6 Cosmology
   Discussion of concepts about the structure and evolution of the Universe.

81 Astrophysics of Stars and Nebula
   A survey of our knowledge about stars.

82 Astrophysics of Stars and Galaxies
   Evolution of stars, galaxies and cosmology

140 Stellar Systems
   Properties of star cluster and galaxies

199 Special Studies
   Special studies with an individual faculty member

EARTH AND SPACE SCIENCES

130 Isotope Geochemistry
Theoretical aspects of geochronology

132 Principles of Biogeochemistry
Study of organic substances as evidence for origin of life

190 Earth and Space Sciences Colloquium
Discussion of current topics

200 Introduction to Geophysics and Space Physics
Physics of the Earth and the Planets

EDUCATION

125 Education of Special Individuals
Emphasis on the psychology of individual differences, learning characteristics, application of research and theory to education

MATHEMATICS

113 Combinatorics
Permutations, combinations, counting principles, recurrence relations, and generating functions

115 Linear Algebra
Abstract vector spaces, linear transformations, inner product spaces, eigonvector theory

117 Algebra for Applications
Congruences, fields, introduction to groups

144 Linear Programming
Principles of linear programming, including simplex methods and duality

150 Probability
A basic course emphasizing theory and applications

199 Special Studies in Mathematics
Special studies with an individual faculty member

201 Topics in Algebra and Analysis
Designed for students in mathematics education

202 Mathematical Models and Applications
Designed for students in mathematics education

V. Sabbatical Activities and Personal Observations

I must admit that I was wrong about education courses. I had never taken any, and I thought that they were concerned primarily with classroom management, and lesson plan formulation. I was greatly mistaken. At UCLA, education is a multidisciplinary program, involving philosophy, psychology, sociology, history, economics, management, governance, administration; even mathematics and statistics. In fact, the UCLA Graduate College of Education offers fourteen majors. There is no undergraduate program offered. The best way I could describe the beginning program in the Graduate School of Education is as a sort of combination between an M.B.A. of Education, and a Systems Engineering degree in Education. Since I plan to spend the rest of my life in the field of education, I realized that many of these courses would be very beneficial to me.

An unexpected result of my leave was what can best be described as a weight problem. During my first quarter, I had five classes in seven buildings. (Lecture and Recitations were hardly ever in the same building.) This required walking between four and seven miles a day. In fact, I lost twelve pounds during the quarter!

During Winter Quarter, I caught a cold which just wouldn't go away. When I went to a specialist, I was amazed to learn that I needed surgery to repair a broken nose. (Most probably broken during my childhood!) I arranged this surgery to occur between the end of classes of UCLA, and the beginning of Summer School at MSAC. (How many instructors do you know that are dedicated enough to wait for vacation to have surgery done?) Although the operation was minor, it required that I stay off my feet for many days, and "take it easy" even longer. As a result, I regained all my lost poundage -- with interest!

I'd like to recount a rather amusing incident. I had to obtain permission to enroll in his course from Dr. Arthur Cohen, who was introduced to me as the world's leading expert on Community Colleges. When he found out I was from MSAC, he asked if I knew Marie Mills. When I told him that she was our second president, he told me that he recalled a meeting he had with her some thirty years ago. She introduced herself by saying that she was from "Mt. SAC, a small college in a beautiful valley circumcised by hills."

Many other of my professors had also heard of MSAC, but most of these learned of the school by way of its excellent Relays each spring. Some remember MSAC because of the high quality of students transfering from there to UCLA.

I again want to express my deepest and sincere thanks for leave to pursue advanced course work at UCLA. I can truly say that I enjoyed every minute of it. In fact, this year at UCLA has to rank as one of the three happiest periods of my life, along with courting my wife, and my National Science Foundation Fellowship at the University of Arizona.

I really did relish my program, and I did profit greatly from it. In fact, I would like to do it against whenever the opportunity should arise. I doubt if others would share my enthusiasm.

The only unpleasant aspect was the commuting. I had to travel almost a hundred miles a day. And in order to "beat the traffic," I would get up before 4:00 a.m. and leave the house by 5:00 a.m. in order to arrive at UCLA by 6:00 a.m. Usually, I would not leave the campus until the Library closed at 10:00 p.m.

Here is a list and brief description of the fifteen courses which I took, along with some comments. (However, the quality of comments should not be equated with the quality of the course.)

Astronomy 10 -- Astronomical Observing, Professor C.D. (Tony) Keyes

My strong background in physics and math ideally prepared me to teach Astronomy. However, I used this Sabbatical to remedy my greatest deficiency: laboratory experience. This course surveyed the current state of the art in the usage of modern instrumentation and counting techniques in astrophysical studies. Most of the course was devoted to stellar measurements. However, there was some work on lunar studies, constellation identification, and cosmology.

Before each experiment, Dr. Keyes gave a half hour lecture concerning the hardware, its use, the astronomical objects being studied. These introductory lectures plus the labs following were marvels in their completeness, yet succinctness. Everything you needed was there without any distractions.

History 3 -- History of Science, Professor Robert Westman

Much to my delight, I found that this title was a misnomer; the course was actually a history of astronomy from prehistoric times to the age of Newton. This was perhaps my most demanding course. It required mastery of five (5) books, as well as hundreds of pages of printed handout material. Also required were a midterm, a final and three (3) term papers. However, the instructor was a master, not only of the history of astronomy, but also of the principles of good teaching, and student inspiration. His was the first course that I've ever taken when at the end of the final lecture the students applauded!

Professor Westman was most generous with his time. I spent about an hour each week talking with him in his office about the history of astronomy. He shared much of his knowledge with me. I shall miss him.

Math 370 -- The Teaching of Mathematics, Professor Larry Schoenberg

This course stressed new methods of teaching mathematics. Also, emphasis was given to including more applications of topics of current interest. Some attention was devoted to present and potential capabilities for television and computer assisted learning. The instructor, Professor Larry Schoenberg, was notable in that he is a former high school mathematics teacher. However, he established such a reputation in teaching, innovation, and motivation, that he was invited to come to UCLA as a visiting professor to present his methodology to selected groups of mathematics teachers.

In this course, Professor Schoenberg stressed applications of math to current problems. For example, you should not show students how to use logarithms to perform calculations in arithmetic. (That is what logs were used for before Pearl Harbor!) As a much more relevant example, he showed how logs are used to explain the Richter Scale which are used to measure earthquake intensity. Each week, Professor Schoenberg gave as an assignment: Find some article of interest in the daily newspaper that you can use in class to illustrate a mathematical principle.

Psychology 110 -- Learning and Memory, Professor John Houston

This was my first psychology course. Since I am devoting my life to teaching students and, hopefully, helping them learn, I felt that I would benefit greatly by studying how the mind works and how learning occurs. Not only did this course fulfill both of these expectations, but it also provided many insights into what I as a teacher could do to facilitate the processes of learning, organizing, and remembering.

This course had two pre-requisites, neither of which I had taken. Moreover, I'd guess that my Sabbatical Committee might not be very receptive toward my taking such an entry-level course in an outside field. Thus I made the course unnecessarily difficult. However, I judged correctly; it turned out that this course would be quite useful to me as a teacher. Consequently, I was willing to put in all the time needed to study background material. I might add that the textbook, which was written by Professor Houston, was a gem. It was very well thought out and very carefully written, making it both pleasureable and useful. It was the ideal supplement to Professor Houston's lectures.

Psychology 134 — Educational Psychology, Professor Wendell Jeffrey

An excellent course taught by an excellent teacher. Professor Jeffrey did a wonderful job of interweaving theory and practicality. In fact I don't see how anyone could have done a better job of correlating teaching procedures with the psychology of learning. Moreover, his choice of textbook was excellent. I have read it cover to cover five times, with much reflection and meditation, accompanied by much underlining and marginal notes. I will learn more in my further study of this annotated text.

Education 125 -- The Education of Exceptional Individuals, Professor Cindy Bernheimer

In modern educational jargon, handicapped students are referred to as exceptional. This course included study of the following groups:

1. Blind and visually impaired
2. Deaf and hearing impaired
3. Crippled and motion impaired
4. Learning disabilities
5. Psychologically impaired
6. Mentally retarded
7. Culturally disadvantaged
8. Economically disadvantaged
9. Mentally gifted

The course discussed cause, diagnosis, and medical and/or psychological treatment of each (where applicable). We also studied their special problems and needs both as people and learners as well as both psychological and pedigogical techniques to better facilitate the learning process.

The instructor was truly remarkable. Twice each week, she gave a two-hour lecture which completely enumerated, summarized, and explained all of the main points of the lesson — as well as most of the other materials, all without reference to text or notes. She would not only recapitulate what the book said, but also call upon her extensive personal experience to reinforce or contradict the author's views.

Education 206 — Introduction to the Philosophy of Education, Professor David Ericson

In this course, we studied the similarities and differences of the principal philosophies of education, from the liberal education of ancient Greece to various viewpoints of the 20th century. There was extensive reading and discussion of original papers by such luminaries as Adler, Aristotle, Bloom, Dearden, Dewey, Hirst, Passmore, Peters, Piaget, Ryle, Scheffler, and Wilson. Attention was given to analysis of educational goals, objectives, contents, methods and values. I found this course rather difficult because I had not had the pre-requisites. However, Professor Ericson was most helpful and encouraging.

Education 209B — Issues in Higher Education, Professor Burton R. (Bob) Clark

This was a delightful, yet very informative course. During the ten weeks, there were eight guest lecturers. Each gave two hours of lecture which covered some facet of the field of higher education in which the speaker had considerable expertise. Especially valuable were the reams of reading material passed out during the course. Professor Clark did a very good job of choosing the right topics to present an across-the-board view of educational problems, and of choosing appropriate speakers to present these topics. Moreover, Professor Clark blended these ingredients to produce a course, not just a collection of topics. A major part of the course was preparation of two term papers. On one of them, I searched my soul for three weeks. It was one of the few papers that I've written that said everything I wanted to say in just the way that I wanted. I benefitted greatly from all the contemplation, sifting, analysis, and ordering of the various factors bearing on the problem. Because of the timeliness of the subject it is included at the end of this report as "Math Anxiety: Its Causes and Treatment."

Education 209D -- The System of Higher Education, Professor Alexander (Sandy) Astin

The course description reads "Identification, analysis, and discussion of current issues, innovations, trends and policies in post-secondary education." This does not adequately summarize the course. This is best done by the title of the textbook, "Achieving Educational Excellence," by Professor Astin.

Throughout the course, the goals were to find what was meant by educational excellence, what factors, influence educational excellence, and what could be done to maximize the good, and minimize the bad. These goals were achieved.

Education 239 -- The Organization and Governance of Educational Systems, Professor Carol Mock

The hypothesis for this course is that educational institutions are best studied as complex organizational structures. The course studied the three basic views of organizations as Systems and Rationale, (a set of formal rules, minimizing the role of people); Natural, (an interaction of people, minimizing the role of formal rules); and Open, (analygous to the operation of a computer). It also shows how these three fundamental approaches can be combined to alter their effects. It then showed why educational systems differ from all these idealizations. It was also shown how these constraints affect the goals, environmental relations, governance of structures, operational processes, and decision making in different kinds of educational organizations.

Among all the schools offering a course in Organizational Systems, one book is most frequently used as the text; Organizations, by W.R. Scott.

I am enclosing at the end of this report a letter from Professor Scott graciously replying to my letter to him, pointing out five errors that occurred in his text.

Education 249B -- Seminar on Institutional Research and Program Evaluation, Professor James Trent

Everyone wants to have good programs taught in a proper learning environment. However, how can you tell whether you're succeeding? This course focused on methods used to analyze and evaluate a program. I have never taken a course in which the instructor seemed less a part of the problem and more a part of the solution! Everyone in the class had the idea that he or she was working on his/her individual problem, and that Professor Trent was at his or her side giving advice and assistance. He was always "us," never "them." Many times, I've gone over his entire program of class management again to analyze what steps he took to achieve this remarkable intellectual camraderie.

We devised and studied procedures to obtain quantitative data as to how well programs were working. These results could then be used to modify, improve, or perhaps even replace, a program.

Education 251 -- Seminar in Science Problems in the Philosophy of Education, Professor David Ericson

If anyone had predicted the effect of this course on me, I would have ridiculed his claim: that I would learn an extremely basic principle of physical science in an education course, taught by a philosopher. However, this is just what happened. An intensive study initiated by developments in Professor Ericson's course enabled me to come to the realization that the age of "the scientific method" is over, and has been for several decades. The complexity of modern research is so great, that all major programs are model-driven, not result-driven. That is, you can no longer summarize experimental findings to produce a theory. Instead you must first decide what is the nature of reality, then conduct experiments to prove (or disprove) your viewpoint.

Thus, after twenty-four centuries, we have gone full circle from the inductive methods of Aristotle back to the deductive method of Plato. Since this result is so basic, and perhaps unexpected, the paper is included at the end of this report for your consideration.

Education 261D -- Seminar on the Community College, Professor Arthur Cohen and Professor Leslie Koltai

These two made a wonderful teaching team: Dr. Cohen is a (full) Professor of Education at UCLA, and Dr. Koltai is Chancellor of the nine-campus Los Angeles Community College District. They would alternate giving lectures. After the one had finished, the other would say "As a theoretician (Cohen), I disagree...", or As a practitioner (Koltai), I disagree...,". The interplay and contrast of ideas was most educational. Dr. Koltai gave one three hour lecture over the subject of how and why we teach that I can only describe as inspirational. He has the very rare gift of talking about personal experiences without sounding sentimental, and talking about personal triumphs without sounding boastful.

Education 312 -- Principles of Instruction, Professor Madeline Hunter

Professor Hunter is indescrible! Almost all professors of education, and most practicing teachers in California know her by reputation. Indeed, many of these teachers had used her books, or attended institutes given by her. In fact, this course which I took with her met on an unsymmetrical schedule because she was often out of town to give special teacher training courses. (One weekend, she was retained by the Department of Health, Education, and Welfare to give an all day institute for teachers on a Saturday in Seattle, and another all day institute Sunday in Chicago. (The latter attended by four thousand practicing teachers.)

Madeline, as she likes to be called, is truly outstanding. She has spent years analyzing the learning process, and the part that teachers, books, notes, blackboards, and lectures, play in it. She has dissected the overall process into what steps the teacher should take in what order to maximize learning. She also has lists of do's and don'ts.

I probably learned more about teaching in one of her lectures that I did in a year in actual teaching experience. I must say that much, surely not all, of what she presented, I had already figured out "on-the-job." However, it had taken me some twenty years to do so. Her course highlighted much of this learning in ten weeks! I wish I had been fortunate enough to take her course in my youth. I cannot praise her adequately.

Education 440 — Administration of the Instructional Program, Professor Donald A. Erickson

This course examined some goals of education, some problems that arise in meeting these goals, and some measures that can be taken to eliminate, or at least decrease these problems.

I found the instructor's viewpoint very refreshing: there are no panaceas for instructional success. Strategies must differ from situation to situation. The teacher must not seek a universally applicable solution, but must select from a broad variety of approaches available for the improvement of instruction and learning.

I thought Professor Erickson's conduct of the class was — what's a word to mean better than superb? He shared with us his actual teaching experiences over his illustrious career to illustrate how certain particular situations require treatment other than that given "in the book." His brilliance is attested to by the fact that he completed the Ph.D. program at one of the best schools in the world, the University of Chicago, in only two years!

Professor Erickson was generous enough to meet me for dinner before class some half a dozen times. The amount of information that I acquired in these sessions was comparable to what I received in class! Truly a remarkable teacher and motivator.

I also found time to audit two courses. I would have liked to have taken them for credit. However, "Maximum" workload at UCLA is eighteen units. Since I had to get special permission to take twenty units, I felt that I would not be allowed to take twenty-four.

These two courses were:

Engineering 14 -- The Science of Engineering Materials, Professor Ming-Feng (Jim) Yang

This was a study of how both the macro and micro structure of materials (metal, ceramic, glass, polymer, and composite), affected their properties, (strength, rigidity, ducility, impact resistance, thermal conductivity, electrical conductivity, etc.

Honors Collegium 41 -- The Origin and Evolution of Life and Humans, Professor J. William Schopf, Department of Earth and Space Sciences, and James R. Sackett, Department of Anthropology

Professor Schopf taught the first half of this course which described how life first appeared on the earth. He also went into the chemical and biologicval phenomenon that made possible the occurrence of this life, and then its diversity. (Professor Schopf is Director of the Origin of Life Institute at UCLA.)

Professor Sackett taught the second half of this course which traced the evolution of this life to its human state. However Professor Sackett did "lie" to the class once. He began with the words, "I'm going to give you a boring lecture today." He then proceeded to give one of the most interesting lectures that I have ever heard! Since his subject was Stone Age Archaeology, you can imagine the man's teaching competence. When I mentioned this great lecture to my daughter (UCLA, 1983), she said "I bet that was Sackett. I had him for Anthropology and he was one of my best teachers."

The former course will be useful for applications in some of my math courses. On the other hand, the latter will be useful in my Astronomy course, where the origin of life is discussed.

I would like also to mention four significant aspects of my education at UCLA that I did not foresee before I started:

1. The seminar Program (Outside Speakers)
   I was fortunate to hear a very wide variety of speakers, from Nobel Laureates talking about astronomy to Henry Winkler (The Fonz), talking about how to make it in show business; from rabbi Mayer Kahone, talking about what it means to be Jewish and militant (in a speech punctuated by demonstrations by opponents in the audience), to Joan Rivers, talking on what it means to be — well -- Joan Rivers.
2. The Departmental Seminar Program
   Most departments -- there are 122, offered seminars every other week. I was able to attend seminars covering a wide range of subjects. These included both reports of new findings as well as synthesis of previous results.
3. The Library System
   UCLA has nine different libraries which give access to 5.62 million books, indexed by computer.
4. The Quality of the Student Body
   According to the California Master Plan only the top 12% of high school graduates may apply directly to UCLA. The ratio of applications to admissions runs about 3 to 1. Consequently, all students are well qualified, and classes progress at a rapid rate. I learned much in discussions with other students outside of class.

   Moreover, most of those enrolled in the graduate courses which I took were full-time teachers. We were thus able to share and compare many problems, strategies, and techniques.

   Each of these tended to broaden, deepen, and enrich my educational experience at UCLA. This was one of the most educational productive, yet also one of the happiest times of my life. I can never adequately express my gratitude to all those at MSAC who made my sabbatical leave possible, or to all those at UCLA who made it so educationally profitable, intellectually stimulating, yet totally enjoyable.

APPENDICES

MATH ANXIETY
Its Cause and Treatment

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Issues in Higher Education
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John Burns
May 1987

DEDICATION

This paper is dedicated to the Greek philosopher, Diogenes, author of the statement, "The foundation of every state is the education of its youth," and to our noble and fearless leader, Governor George Deukmajian, in the hope that he will read it someday.

SETTING THE STAGE

Part I: Why Johnny Can't Read

An Environmental Blueprint for Failure

(This might well be skipped by serious scholars.)

The lack of mathematical ability is but one symptom of an across-the-board lack of ability. As SAT scores have shown, there has been a general decline in achievement compared to ten, twenty, and thirty years ago.

Several reasons must share the blame. I'll call them environmental.

1. There has been an enormous increase in the number of single parent households. This is usually caused by divorce, which is curently not far behind marriage in frequency; or especially in the last 15 years, in failure to be married once Junior is on the way. Raising children these days is a hard enough job for two parents; it's just about overwhelming for only one.
2. Even when there are two parents, economic necessities usually require that the mother work. Studies have shown that children learn to read best if they are read to - often - before they start school. Most commercial preschools cannot do this.

   A new phenomenon is the generation of latch-key kids. Children who return from school to an empty house while both parents are at work. When one subtracts the amount of time the mother spends preparing for work, getting to work, at work, and worrying about problems related to work, it's not surprising that she is usually too busy to talk with her children about school and school problems. She is often too busy, or too tired to help them with their homework.
3. Another bad aspect of the previous two problems is the lack of a parent available as a role model. Most of the best students come from families where they grew up watching their parents reading or studying frequently.

4. The one-eyed monster: Today's children are close to the first generation of collegians who grew up in an environment saturated with TV. There are two aspects of this. Both of which retard learning:
   a. The great majority of the material viewed on TV is intellectually worthless, some is even harmful.
   b. Between language comprehension in childhood and the completion of the formal education, the average child spends over 14 hours a week watching TV. Since Today's days are not 26 hours long, that means that there are two hours a day which is not available for the pursuits engaged in by previous generations. While previous generations "played," modern children sit and watch. Surely even the best of today's students take a few hours from study time to watch the playoffs (pick your sport), the special (pick your event), or the weekly show (pick your sitcom or soap).
5. Today's collegians are also close to being the first generation to have their own cars since legal driving age - or before. This requires a job to pay for them and their upkeep, all of which is time not available for study. Usually these cars are used, which means they need repairs frequently. This then takes more money, which takes more time. Some drivers are do-it-yourselfers, another time consumer. Then cars are a great invention, but they have their bad points too. They are really great places for those scourges of American society: Sex, Drugs, and Rock and Roll!
6. This is also the first generation of collegians to have lifelong access to a bewildering diversity of drugs. Surely math does present some challenges under normal conditions. For the student "high on drugs," or "mellow" on alcohol, these challenges are just about insurmountable.
7. The general deterioration of the intellectual climate in America: Today's kids don't want to be doctors, lawyers, or merchant chiefs; they want to be rock musicians or pro basketball players. Last year a sportscaster was fired for saying that the basketball team of a certain college had an IQ of 120 - collectively. He may just have been close to correct.

8. The decline of money for educational purposes:
   a. Federal: Each year we see a decrease of money allocated to scholarships, student loans, professorial research, and program innovation.
   b. State: Twenty years ago California ranked first in educational funds per student; today, it is 29th.
   c. Locally: Probably nothing has hurt the educational system in California more than Proposition 13. The impact of this was especially severe on the community college system, which currently receives less dollars per student than do the elementary schools. To quote that renowned educator, W.C. Fields, "Money isn't everything, but it sure beats whatever is in second place."
9. The erosion of the desire to excel: In the last twenty years the role model has shifted from Vince Lombardi's ("Winning is not the most important thing; it's the only thing"), to today's lonely, misfit loser as portrayed in any Sean Penn movie.
10. The rise of violence as the solution of any problem: Last year over 200,000 teachers were attacked physically by students. A professor at CSU Northridge was murdered by a graduate student last year. Another was murdered by a graduate student ten years ago at Stanford. Two years ago, a teacher in a Chicago school was raped while her third grade class watched.

    These problems are probably more widespread, and even worse, so deeply-seated in the fabric of our society, that no immediate solution -- not even a strategy or solution is in sight.

Three additional changes occurred during the last decade at the community college level which were especially significant in California:

1. The unionization of faculty and collective bargaining.
2. The change of most colleges from the Departmental Structure, (in which the chief, perhaps only, intermediary between faculty member and administration was the department chairman); to the Division Structure, (in which a new

intermediary arose in the person of the Division Dean, (accompanied the de-emphasis of the Department Chairman in both power and renumeration).

3. The replacement of the voters of each local school district by the legislators in Sacramento as the major source of funding approval or denial.

The results of each of these changes was profound, ongoing, and not completely assessable as yet. Even more sobering to consider, all three of these changes occurred in a period of just a few years!

Part II: Johnny is Versatile.
He can't do math either.
And, he <u>can't</u> do it even better.

Assume a Martian were to visit Earth, landing, let's say, in the wilds of Westwood. He/she would probably be able to blend in with some element of the native population. Since they all attend someplace called "College," he would want to attend also. If he asked them what was the easiest major, he would get many opinions and long arguments. However, if he asked about the hardest major, the answer would be quick and unamimous: Math! If he were a scientifically inclined Martian, he could to to the Techs (Cal or Trade); if he were a devout Martian, he might go to Loyola, Whittier, or Cal Lutheran. If he were slightly retarded, he might be more at home across town at University Park. However, in every case, his answer to the search for the hardest major would receive the same answer: Math. This gives rise to the question; is this because of the teacher, the book, the inherent nature of the subject, or is it "all of the above?"

Let's explore some possibilities:

A. Instructor Related Problems:

One of the greatest invitations to disaster was the <u>K-14</u> Lifetime <u>Credential</u> which, fortunately, no longer exists in California. Under this credential, anyone who graduated from college and who had satisfied the requirements to teach in any field, say English at the elementary school level, was legally qualified to teach <u>any</u> course up through the second year of college. That meant that if this teacher were laid off from his job teaching junior high school political science, he could be assigned to the local community college to teach, say, math, Portugese, nursing, and/or modern dance.

Although California has discontinued giving these credentials, other states still do award them. This should stop immediately. However, California still has over a million such credentialed teachers. The California State Board of Education should take action to prevent these teachers from teaching in fields in which they don't have even a rudimentary knowledge. Regardless of previous credentialing, I would require that if a teacher had no major or minor in a field, and has not taught in that field within say, the last ten years, then he should be required to pass an exam in that field, or take coursework before being allowed to teach in such field.

For many decades, the flower of the American system of higher education was the Liberal Arts Colleges. Molded in the British tradition, they all required several years of math. Many math teachers do not know why. Math was required not so that students would learn math. Instead, it was considered the best training in logic, deduction, analysis, and cognitive thinking in general. This is the classical Greek idea that the highest goal of education was to develop the ability to reason, (not to write on empty slates). The process, not the product was emphasized.

Do we really feel that the English major, even with his Master's in Shakespeare will accomplish any of the former, let alone much of the latter purpose? Only someone who has had an extensive enough exposure to math to have an effective overview, and who has studied parts of math intensely enough to have complete comfort with the parts he will teach, can be effective as a teacher. The student gains almost nothing from the teacher whose math training is so deficient that his class preparation consists of trying to reconcile the problem sets with the answer book. Even worse, the greater the talent of the student, the more he will be frustrated by such a teacher.

Proposed Solution:

1. Eliminate across the board, certification beyond the K-8 range, as has been already accomplished in California.

Increase Teacher Compentency

California has taken a step in the right direction with C-BEST, a test in minimum teacher competency. One of my years' teaching at Cal State, I was put in charge of giving entrance exams for our class, Modern Math for Teachers, to filter out those whose background was so poor that they would slow down the class. This test required a knowledge of math up to only the 6th grade level. It took a grade of only 56% to pass. One of those who failed the test had spent the previous three years teaching 5th grade in a public school in California. Can you imagine how much he could impact his students?

Proposed Solution:

1. Require special math courses as explained above.
2. Require that all prospective teachers pass a general test, somewhat like the C-BEST.
3. Tighten up the program which awards Emergency Credentials.

During my Cal State years, I had in my classes many such people. Without exception, they were extremely deficient in their mathematical knowledge. To teach a subject, the very first requirement is that you know it. I will not go so far as to say that if you know it, you can teach it. I would say that just the opposite is true. If you don't know it, you can't teach it. Moreover, some of these people had been going through the motions of teaching math for up to five years, during which time they were unsuccessful in their off and on attempts to pass this one course!

Proposed Solution:

1. Restrict greatly the number of Emergency Credentials awarded.
2. Require that everyone receiving an Emergency Credential has completed all baccalaureate and credential requirements in both math and English.
3. Take a long hard view at the requirements to obtain any teaching credential. Much of this would not apply to California which already requires a Bachelor's degree in an academic subject before one can even be a candidate for a credential.

Proposed Solution:

1. Require that teaching candidates complete at least one Madeline Hunter type course. Too many schools think that the ability to make out a lesson plan, and a seat chart make one a teacher.
2. Examine the academic environment of the schools awarding teacher certification. Could there be some significance in the fact that, nationwide, about 50% of the people who start college eventually finish; whereas the figure for prospective teachers is over 80%?

3. Require an academic major in a department certified by the accrediting organization in that discipline, (not merely by a state board).

Teacher Recruitment Should Begin at Home. Two years ago, Los Angeles Unified School District went to Canada to hire teachers. They got some 300. Last year, they hired 500 teachers - sight unseen - in Spain. The reason given was that there were no competent teachers available in LA. Don't you feel that American teachers could do a better job teaching students in LA than teachers from a strange environment. I view with disdain the explanation that there were no teachers available. I know too many credentialed teachers who are working as secretaries, airline stewardesses, waitresses, and clerks. What the district should have said is; "We can't find any competent teachers in LA for $18,000 a year." Shame on them! (By the way, could it just possibly be that this whole scheme has some tones of union-busting?)

Proposed Solution:

Require that all newly hired teachers:

1. Be citizens who can speak English; or
2. If aliens whose native language is English:
   a. Require a minimum of three years experience in teaching at the same level.
   b. Add to that requirement, an acceptable degree and teacher certification, not Emergency Credentials.
3. If aliens whose native language is not English, require:
   a. Passage of a fairly rigorous examination to insure the ability to speak English fluently. This should include testing over both grammar, and pronunciation.
   b. Require at least five years teaching experience at the same level.
   c. Require acceptable degree and teacher preparation by American standards (not those in their home country). Again, real credentials, not Emergency.

Note well:

Many well-intentioned people scream discrimination as soon as they see the name of a minority. The above proposals are not discriminatory! The subject under discussion is the competence of the teacher in English, not his place of birth. No matter how brilliant he may be, how well he has mastered his subject, or how skillfully he presents it, the student will not benefit unless he can understand what the teacher is saying. This author, attending college shortly after the war, was exposed to a large number of Germanic and Chinese accents. So much time was spent trying to figure out what had been said! Not only could this time have been better devoted to studying the meaning of the ideas, but also the delay took the edge off some very elegant lectures.

Our students deserve the best. (Remember, native Americans also need prove their competency in the English language - probably at a much higher level - in the C-BEST.)

Instructor Related Problems:

2. Math Specific

My opinion is that one major part of the problem is teacher anxiety. 83% of the elementary school teachers in Los Angeles have degrees in either English or history. Hence, when these people teach math they are quite far from their preferred field of endeavor. As we have shown above, math is different from any other subject taught at the elementary school level. It is quantitative, not qualitative. It is exact, not approximate. If 3 is the right answer, 2.9 is the wrong answer. Probably the difference is that the nature of math bothers the teacher. It's not that he doesn't know math, it's just that the philosophy and nature of math are starkly different from those in his own discipline.

Hence, even though he may explain the material well, and do all the problems correctly, he is never totally comfortable with math. He just doesn't really like it. Consequently during his math classes, the teacher conveys some of his uneasiness to the class. They don't know just what is wrong, but they sense things just aren't the same. He talks faster, or slower; he doesn't like to be interrupted; he discourages questions; he never does any problems that aren't in the book; he tends to cut the class short; he doesn't ever smile; or crack jokes in math, etc. Sometimes, before class, he thumbs through the teacher's manual or answer book. Although the students may not even realize it, they pick up the attitude that things are different in math class or math is an "up-tight" subject. Math is definitely not a "fun" subject.

Proposed Solution:

a. Require prospective elementary teachers to take courses covering all the concepts that they will teach to develop a broad understanding of the material and its inter-connections.
b. Require that these courses be passed with a grade of B or above to develop a thorough enough knowledge to develop complete comfort with the material to be taught.

The nature of his work requires that the math teacher live in a world unknown to most; a world of linear transformation, Riemann integrals and (hopefully) uniformly convergent series. Because the math teacher must live in this world to get his degree, and to do research, many question whether he lives in the real world.

Proposed Solution:

My personal feeling is that math teachers take themselves too seriously. They seldom smile, never laugh, and always have an extremely knowing expression on their face. This is what I call "The Great Stone-Face Syndrome. I have been quite successful as a teacher by breaking this stereotype. I wear my "Wizard suit" on Hallowe'en. I always comment on the current troubles of the Rams, Lakers, and Dodgers. I always know at least something of what it says on Page 1 of the Times. I don't think my astronomy classes will ever forget that I brought cookies and cakes to class last year for a party on Edmund Halley's Birthday! Then during the NFL Playoffs, I wore my Bears "Refrigerator" Shirt to class to bait all the Rams fans.

I feel that such displays of deliberate moronic tendencies help to establish me with my students as "us", not "them." As such, they actually respect me more, and learn better from me.

A word of caution: Don't go too far!

Many students perceive the math teacher as so unapproachable that they are "too scared" to ask questions.

I think that I am rather unusual among math teachers because I don't think that math is the most important subject for a person to take. I think English is; at least for an American. Every math book has a couple of chapters of word problems. So do all courses in what is actually applied math, but is known under the names physics, chemistry, engineering, econometrics, educational statstics, etc. If the student is not quite fluent and rather fast in his use of these symbols, he'll <u>never</u> make it through math.

One term I had a student in my algebra class who seemed rather intelligent and hardworking, but who still was carrying a D average. It wasn't until I sought him out, and began talking with him that I realized he could not hear well enough to understand these word problems. E.g., he did not know that A is three more than B meant the same as B is three less than A. Nor did he know that other ways of expressing the first realism were; A exceeds B by 3, A is 3 greater than B, if three is added to B, A will result; A - B = 3.

I feel that such displays of deliberate moronic tendencies help to establish me with my students as "us", not "them." As such, they actually respect me more, and learn better from me.

A word of caution: Don't go too far!

Many students perceive the math teacher as so unapproachable that they are <u>"too scared" to ask questions.</u>

<u>One solution:</u>

On the first day I announce in class that I will give a five-point bonus on the next test score to anyone who asks a question over that day's assignment that I cannot answer. (This not only breaks down fear of asking questions, but it is also a stimulus to keep up to date.

While we're on the subject of schemes, to increase the student's involvement, here is another one -- which I usually, but not always -- give as an extra credit exercise: "Make up what you consider a good chapter test over the current chapter. You will be graded over the quality of the individual questions as much as how thoroughly your test covers the high points of the material of that chapter." This forces the students to not only study the chapter but to organize it to obtain complete coverage, and analyze it well enough to determine what the high points are.

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If the student understands the material well enough to concoct his own problems to test it, he probably has a pretty good knowledge of it!

Moreover, the mistakes made here often give the instructor valuable feedback as to what he needs to stress, move and explain better, both now and the "next time around."

Warning: Don't use this technique too often, nor for just any chapter. Make sure that the chapter you assign is easily separable.

A math teacher must have <u>specific training in math</u>. It is acceptable to have someone who is not trained in math to teach math at the elementary level. Perhaps it is also acceptable to have a person with such a lack of specialized training in math teaching at the junior high school level. This would depend on the quality and quantity of math, the teacher's background in math, the level of the classes, and the preparation of the students. However, a person who has not been specifically trained in math should not be assigned to teach math at the senior high school level.

Although a person without such training could show how to do the problems, and could perhaps explain the theory, he could not convey the truth, the beauty, and the excitement of mathematics. Math is <u>not</u> a series of isolated techniques which are indiscriminately applied to a variety of unconnected topics. The solution of every problem should involve three parts: The anticipation of the confrontation, the thrill of the chase, and the beauty of the solution itself. The presentation of the problem should, in the idealized case, strike the same note as is felt before the start of the football game, the beginning of the race, the curtain going up on the play, or the climb up the mountain. Sure, it's going to involve some work, but it'll be fun too. It's not a question of whether we can do it, but of how well we can do it.

Proposed Solution:

1. Require prospective secondary teachers of math to have at least a minor of 20 or more hours of math courses.
2. Require that those courses be chosen with the advisor to form a coherent, useful pattern, not just a cafeteria-style loading-up.

The student must be shown that each problem is not a separate experience. He must be shown how all the different problems are merely different facets of the same underlying principle. He must be shown how to generalize an entire set of problems into a unified whole. Moreover, he should be trained well enough in fundamentals -- both the manipulations and the logic -- that he does not have to pause to think once he starts. He should have learned the previous material well enough so that all he has to do is choose the correct technique. Then, after the choice is made, the procedure should flow along to completion of its own momentum. This is the way a point guard views stealing the ball, dribbling down the court, and laying it in. Once he sees his opening, the rest just develops from habit. It is not necessary for him to analyze each step. He sees the whole thing as a single event. This is the desired state for doing math problems. Once started, the whole process should be a single smooth thought; not a bunch of bumps and starts like a car with a motor which is shorting out.

How can an instructor possibly convey this mood if he doesn't feel it himself?

This leads directly to the question of what an education should consist of. Probably the best answer is the ability to solve problems. Not just math problems, but challenges of any kind. The teacher, unless he is quite competent in math, misses this point. Unless he really understands math, each problem becomes an end in itself, not just a means to the end of teaching how to analyze, to reason, to think. He should know his math too well, then he can solve the problem effortlessly while devoting his thoughts to explaining the overall picture. If the teacher, due to his lack of understanding, spends his time on checklists, or recipes, or special little rules he has worked out, he has missed the main idea. Mathematical understanding involves the ability to see the whole picture, to generalize, to see how the different problems are mostly instances of the same basic principle. Mathematics involves logical analysis of the problem, determining its basic nature, and realizing how it fits into the overall square of knowledge. It does not involve merely memorizing rules or plugging numbers into formulas. True mathematicians operate at the top of Bloom's pyramid, not the bottom.

This overview, this insight, this mastery result from extensive training and intensive practice. They are not picked up by the casual shopper in the shopping mall schoolhouse. They require in most cases, the training which accompanies a degree in mathematics. Unless the teacher knows and feels those things himself, it is surely impossible for him to communicate them to others!

Proposed Solution:

1. Give salary increments to teachers of math who have worked one or two years full time in industry, or research as mathematicians, (and maybe half increments for those who have worked as engineers, physicists, or statisticians.)
2. Inaugurate a series of mathematics enrichment courses over the summer at cooperating universities specifically designed for practicing teachers.
3. Establish a system of government scholarships and/or salary increments to encourage attendance.

The "Green Stuff" especially impacts on math because math teachers have good prospects of obtaining well paying jobs outside of teaching. During a class at a course in educational psychology, the subject, "Why Johnny Can't Read" was being discussed. The teacher asked, "Would it help to raise teacher's pay?" I was shocked to hear someone with supposedly enough intelligence to gain admission into UCLA, and to survive long enough to achieve upperclass status answer, "No, that might attract the wrong people into teaching. They might just teach because they want the money, not because they want to serve others." (I hope he doesn't ever break a leg; he would be surprised when he got to the hospital.) I have also heard that many people are now going into stock brokerage, basketball, and guitar playing for the same wrong reason!

I was also shocked a few years ago to hear an ad on the radio that the California State Prison System was looking for "Corrections Officers," high school diploma required; starting salary $26,000 a year. At this time, the California State College System was hiring professors, Ph.D required, for $25,000. Should helping "good guys" build for the future be as important as punishing "bad guys" for the past? (No, Virginia, there is no correction occurring in correctional institutions!)

Now that we have established the fact that an assistant professor of engineering at a California State College (which requires now not just a Ph.D., but usually post-doctoral publications, experience in a specialized field, and often a minor or masters in a different field), starts at $25,000 a year, let's reflect on the notion that an engineering graduate of a California State College can start at about $30,000 a year. Thus we seek a person with enough intelligence to earn a Ph.D. who is stupid enough to work for such a low salary. Are these really the kind of people we want to attract? Checkers at Safeway now earn more than $25,000 a year. And they didn't have to pay $13,000 tuition to Stanford for four years, nor work 80 hours a week for the next six years to qualify.

Note 1: One-fourth of California teachers leave before serving five years. Do you really think that these are all the least able teachers?

Note 2: Have you ever known a tenured associate professor who got a job selling real estate during his income-less summer, and made so much money that he couldn't afford to go back to teaching? I have.

Note 3: You must admit that there is something wrong with the system if beginning school bus drivers are paid more than beginning teachers. (L.A. County School District.)

Note 4: There must be something very much wrong with any system that has to base its school financing on selling tickets to a lottery.

Moral: Many present teachers will leave; most will stay regardless. However, if we want to retain our best teachers, and attract even better ones, we must have an increase in salary. (At present salary levels, most teachers will not make in their whole lifetime what Kareem, or Magic, or Fernando, or Pedro, or Eric earn in one year! Even Lasorda and Dieter (remember him?) earn more in a month than a teacher does in a couple of years. Do you really think it can be said today, as it was in the past, that the top students go into teaching?

Not only are teaching salaries lower -- much lower -- than the same people could earn in industry, but also much lower are the fringe benefits: medical insurance, retirement, travel. How many colleges give Christmas bonuses, or stock plans?

Proposed Solution:

In view of the present attitude in this country towards teachers, it would seem that the only hope may be to begin exploration for gold and/or oil on school district properties!

SUMMARY OF PART A

Treat the teacher with at least a veneer of respect. This year, the California State School Board extended the school year by one week. No one asked the teacher's opinions, or even thought of increasing their pay for this increase in hours. In no other industry would such conduct be accepted. This is the latest of countless cavalier actions. After all, if the board treats their teachers as cattle for long enough, they'll start behaving as such.

Note well: School board statisticians, in their ignorance, bandy around data such as only 8% of our teachers resigned last year. They don't realize that it's always the most capable that leave.

Proposed Solution:

Getting the general population to respect teachers may take a while. However, at least their own school boards should respect them. Some steps in the right direction would be:

1. Increase teacher salaries. (In America, higher respect seems to accompany higher salaries.)
2. Relieve teachers of much of the administrative and clerical load which takes too much of their time, and lowers their self-respect.
3. Eliminate much of the warden-like psychology from schools. Competent professional teachers should not be required to sign in every day, or bring a note from their doctor when sick, etc.

B. STUDENT RELATED PROBLEMS

Math is a different kind of course than say sociology, or history, or even economics. It is primarily quantitative rather than qualitative. This does not say that it is better (or worse); merely that it is different. As such, it requires different talents. Probably most students have not used such talents. Therefore, it might be beneficial to have some orientation meetings covering how math differs from other courses, and tips on how to succeed at math.

Surely, the students must be made to realize that while you may learn other courses by reading, you learn math by writing. Probably, if you do not use a pencil, you do not learn. Math, like swimming, is not a spectator sport. No matter how good your instructor is, or how many lectures you attend, you can only learn swimming by jumping into the pool and splashing around. For math, working problems with paper and pencil plays the role of work in the pool.

Trite as it seems, the only way to learn to work math problems is by doing math problems. Nothing works as well as repeated student effort accompanied by teacher guidance and feedback! As I tell my students "If you are so busy working, studying other courses, or dating to have time to write out your math homework, don't worry. This course is being offered next term (and you'll probably be taking it then)!"

Proposed Solution:

Such concepts should be integrated into courses required for teacher certification in math.

One aspect of difference is what I call the "put-down" factor. Suppose a student is taking a course in psychology, or sociology, or political science; no matter what he writes on an essay question, he can say he is partially right. He deserves partial credit. His grade can often be raised by arbitration. Math is different. If the answer to a problem is 3, then 2.9 is wrong. Very often the student will leave out a step in a line of reasoning and get a wrong answer. Often a profoundly wrong answer. However often what the student sees as a slight "overlooking" of part of the problem, or as a slight error in manipulative technique, the instructor will see as a major blunder.

This is hard on the students in two ways:

a. Grade:

The student will often find it hard to get partial credit for the problem. A large number of people graduate without ever working a problem right, but they pile up enough points on partial credit to make it through.

b. Ego:

Suppose the student has worked the problem right except for one small part. When he says the answer is 3, and the instruction says that it is 29, he really feels crushed. Since he didn't even see the piece of logic he left out in solving the problem, he naturally can't see where he made his mistake. He feels that he just can't understand math. When the instructor puts an abbreviated solution on the board, mumbling to himself all the while, the student can neither see nor hear the answer. His ego really drops when he can understand neither the problem nor the solution. It is easy to see how he gets the idea "Math is hard, or I can't do math." Once acquired, this attitude is very likely to stay.

Proposed Solution:

Such consideration for student sensitivities should be incorporated into courses required for certification of all teachers.

I strongly recommend the approach given in Madeline Hunter's "Mastery Teaching." This corrects the student without deflating his ego. This would be exemplified by the exchange: Teacher: "Who was the greatest American hero of the Revolutionary War?" Student: "Benedict Arnold." Teacher: "Well, that's not quite right. If I had said 'Who was the only American general to betray his post to the enemy?', your answer would have been correct. Didn't you mean to say, George Washington, or Light Horse Harry Lee, or John Paul Jones?"

C. SUBJECT RELATED PROBLEMS -- AVOIDABLE

Math involves different study techniques. In most subjects, the student's only homework requirement is reading. However, math is different. No matter how much the student reads his math textbook, it is unlikely that he will really master the material without actually working problems.

The teacher must remind the student frequently of the necessity of "studying with a pencil."

Eventually, the student will ask: "What do you use this stuff for?" "What is its applicability?"

Proposed Solution:

The best response the instructor can give is:

"I'll tell you the answer as soon as you tell me what you'll be doing for the next fifty years. Much of this mathematics will be necessary to understand things which have not been invented or discovered as yet. You can expect to retire fifty years from now. Ask your grandparents to tell you some of the things that didn't exist when they were your age. You'll be amazed how low the list will be.

Easy problems can be solved by using simple tools. Bookkeeping requires only arithmetic; high school physics requires only high school algebra; but college physics requires college calculus, and electromagnetic fields require vector calculus. Most of the simple problems have already been solved. Only the hard ones remain.

Another aspect that makes math harder than many other subjects is what can be called the "Heinz" factor: How tough is the catch-up?"

Assume the student misses, say, a week of classes for either business or pleasure. In English, he has missed all of Moby Dick, but he can read Hamlet with the class, and study Moby over the weekend. In history, he has missed the War of 1812, but he can study with the class about westward expansion, and the controversy over slavery. He can also make up this missed material at his leisure.

Things are different in math. He has missed all of chapter four, and the class is on chapter five. However, when he comes back he can't understand a thing being said about chapter five because all the material is based on chapter four, which he missed. Therefore, he wastes his time sitting in math class and not being able to understand. So he works hard at home and finally finishes chapter four on his own. In the same length of time that was devoted to it in class -- a week. (That in itself is hard to do without a teacher's explanation and guidance.) However, now that he is ready to start chapter five, the class has in this same week, progressed to chapter six. Again, he can understand almost nothing in chapter six because he hasn't done chapter five yet.

Moreover, he learned almost nothing about chapter five during the four hours he sat in this class while they were discussing chapter five, because at that time, he didn't know chapter four, which chapter five builds on. Hence, he must study this on his own, also. However, by the time he has learned chapter five the class is into chapter six. Thus, he is on the treadmill. Once behind, it is very hard to get caught up in math, because math is cumulative. In general, you cannot skip. You have to get "caught up" first.

Proposed Solution:

1. Have the teacher explain this catch-up problem to the students at the beginning of the term.
2. Encourage students who will be out for a long time to send for their books and assignments.
3. Set up conferences between student and teacher, by phone while the student is away; in person, after his return.

The "Frontier-Scout Syndrome" can really confuse the student. Mathematicians must master the techniques to carry out mathematical operations, and accomplish mathematical results. Good mathematicians develop short-cuts. Often the better the mathematician, the more short-cuts. However, these should not be given to the class. Often, without thinking, the teacher will use one of these short-cuts, and completely lose the class. (Usually, if a student asks me about a short-cut, he "knows," I'll tell him to see me after class. Very often, a student's "short-cut" is wrong all or most of the time. It seemed to work on a simple problem -- often a mere coincidence -- but will not work in a general case.) Even if the short-cut produces the correct results, the student seldom knows the conditions necessary for its use. Since short-cuts are usually run through pretty fast over the tops of the mathematical icebergs, the student will seldom either understand, or even remember, these necessary conditions. Moreover, usually the use of the short-cut requires a thorough comprehension of the problem and its side effects. That is, although the short-cut utilizes a simpler technique, it requires a more complete understanding.

Proposed Solution:

Never, never use short-cuts in teaching math -- well, hardly ever.

ORGANIZATIONAL PROBLEMS IN SCHOOL

Many high schools have so decreased their math offerings that students are unable to take four years of math. If each high school cannot provide such instruction, then there should be at least one high school in each district which does offer these courses, and to which eligible students could be transported.

I would recommend that high school also offer a few math electives. (Consumer math and personal finance math don't count. These are essentially courses in bookkeeping, not mathematics.) The normal sequence is elementary algebra, Euclidean geometry, intermediate algebra, trigonometry and college algebra. Surely there could be one or two elective courses -- branches on the tree of math, not more trunk. Examples would be probability theory, statistics, theory of numbers, history of math, non-Euclidean geometry, and linear programming (which is not a branch of computer programming).

Also, the high schools should do a better job of introducing computers into the student's awareness.

Proposed Solution:

Require three years of math for high school graduation. (Real math, not acounting courses carrying math titles such as business math, personal finance math, etc.)

There are too many academic distractions from basic courses. Paradoxically, while high schools have cut back on their offerings of college level courses, such as math, science, foreign languages, they have increased their number of electives beyond reason. Often in an attempt to overcome student apathy or disruption, some electives are chosen for amusement, not for education. How can an English department justify science fiction, various courses in "protest literature," and "soaps" as classics, if they can't offer enough grammar and composition? Of course, there is nothing intrinsically wrong with such courses. Unfortunately, they tend to pull students from the traditional college preparatory courses.

Proposed Solution:

Require more college-prep courses. These should include:
1) Four years of English; 2) Three years of math; 3) Two years of natural science; 4) Two years of a foreign language (which cannot be the language which he speaks at home. We assume he already knows that.)

A VERY WIDESPREAD PROBLEM IS SOCIAL PROMOTION

Since this is a deep-rooted problem, let's begin at the bottom. In Los Angeles schools a student cannot be held back without written permission from both parents. The reason given for this is that it will hurt the child's ego to be held back. I cannot condemn this philosophy too strongly. Promotion is not a social grace, or a right. It represents certification that the student has learned the material covered at one level, and is ready to meet the challenges of the next. To prevent this process can do the child irreparable harm. If the student has not learned to read in first grade, he is not ready for second grade. The second grade teacher is probably too busy to pull him aside and tutor him in last year's work. After a few of these social promotions, the child has risen to the level where the teachers do not have the expertise to reteach all that has been missed along the line. Once the child is in fifth grade, reading at a second grade level, he has effectively left the learning system. He occupies a seat, uncomprehending, his boredom and anger increasing -- a failure to himself, a trial to his fellow students, and a frustration to his teacher. The best place to learn first grade work is in the first grade.

Even if today's students are not brilliant, they are "street smart." They quickly learn that they cannot fail. Therefore there is no incentive to succeed. This starts the long, often irreversible slide to mediocraty. In my opinion, the removal of the teacher's power to fail students is the chief blame for today's statistics that 25% of high school graduates are functionally illiterate.

Two additional problems are engendered by this fail-safe policy. Since academic natural selection can no longer cull out those with the least talent, the classes become progressively more intellectually diversified. Some teachers like "smart" classes, and some teachers prefer "dumb" classes, but no teacher wants a widely diversified class. Not only are different approaches needed to instruct the A vs. D students, but to slow the class pace down to that of the "slow" (euphemism) student will leave the faster student beside himself with boredom. The reverse procedure of speeding up while satisfying the brighter students, will leave the slower bewildered.

Moreover, the clowns and troublemakers usually are among the first to flunk out. However, this "no flunk" policy ensures their presence to enrich the teacher's existance.

Proposed Solution:

1. Each grade level should have a list of the vital points to be mastered in that grade. If the student does not master at least say, 80% of these, he should not be promoted to the next grade.
2. Since reading is fundamental to all school learning, he should never be promoted if seriously deficient in this area.
3. Perhaps there could be special reading classes so that a student could be provisionally promoted in everything but reading. He could then be assigned to a special reading class or tutorial class until he has caught up with his class. If he did not do so by the end of a year, he would not be eligible for another promotion in all but reading.

Non-English-speaking students come in many varieties. This is not saying anything about the successes of different non-English instruction programs, nor their value to the student. The foreign student should be required to learn English before starting school. One of our feeder school districts contains non-English speaking students whose native language is one of 39 different languages. It is much more logical and cost effective to get 39 students to learn English, instead of having each teacher learn 39 languages.

Until my students can understand what I am saying, they cannot benefit from my instruction no matter how good it may be!

I still remember with sadness one of the students that I had many years ago when I was a professor at Cal State University. He was taking my course in advanced calculus. This course spent more time on proving theorems than on working problems. Moreover, in this course the proofs involved the precise meaning of similar terms: dense, dense-in-itself, locally dense, sequentially dense, etc. My student held a bachelor's degree in math from the University of Yokohama. However his English was very bad. I warned him after each test that he was flirting with disaster. Despite much hard work, and a lot of help from the teacher, he got 20% on the final which resulted in his first F. His decisive failure was not due to his inability to grasp the concepts, but to his inability to know what the words meant. I still feel a sense of guilt that I couldn't convince him to drop, and devote the extra time to learning English.

Note: Inability to understand English is not a problem that is unique to the foreign born.

One term I had a student in my algebra class who seemed rather intelligent and hardworking, but who still was carrying a D average. It wasn't until I sought him out, and began talking with him that I realized he could not read well enough to understand these word problems. E.g., he did not know that A is three more than B meant the same as B is three less than A. Nor did he know that other ways of expressing the first realism were; A exceeds B by 3, A is 3 greater than B, if three is added to B, A will result; A - B = 3.

I think that I am rather unusual among math teachers because I don't think that math is the most important subject for a person to take. I think English is; at least for an American. Every math book has a couple of chapters of word problems. So do all courses in what is actually applied math, but is known under the names physics, chemistry, engineering, econometrics, educational statstics, etc. If the student is not quite fluent and rather fast in his use of these symbols, he'll never make it through math.

Proposed Solution:

Require all non-native English speaking students to pass realistic examinations before being admitted to regular classes. Institute special classes to teach them English.

Too many other jobs have been assigned to the schools. In the old days, the public schools were considered to be teachers of the 3R's. Today, however, they have become the conscience of the nation. Modern public schools, besides fulfilling their historical role of teaching the basic college preparation subjects, also are expected to teach driver's education, sex education, gay awareness, AIDS prevention, racial integration, minority sensitivity, religious freedom, birth control, resource conservation, ecology, world unity, arms control, political involvement, first aid, and heaven knows what else. Now teaching any or all of these subjects may fulfill societal needs, but they do not fill educational needs. There are only so many hours in a day. The more non-academic studies you bring in, the less time there is for the basics.

Proposed Solution:

Most all of these courses could be taught by teacher-aide type instructors. Not only would these save the teacher for the traditional college preparatory subjects, but it would be more cost effective in the long run.

We have become a nation of sue-ers. No matter what happens to us, someone else is responsible. In the old days, a man would have too much to drink, hit another man and go to jail. Today? No Way! First, the drunken driver will sue the bartender, who should have realized that he'd had "too many" and not sold him any more to drink. Or he will sue the liquor company, for making the stuff in the first place, or for not putting labels on each bottle, saying, warning: "This product may be hazardous to your health." If that fails, he will sue Ford for not making bigger brakes, or the maker of the victim's suit for not putting in a collision-proof lining. The same sort of nonsense is allowed in the school. If the student doesn't learn, then the parent can sue the school or the teacher, for inattention, incompetency, or discrimination. Of course, if the teacher were to keep the student after school, the parents could then sue for these things. How can a teacher think effectively about his courses and students if he's always worrying about protection.

Think of the plight of a kindergarten teacher when one of his charges wets her pants. No matter what he does, he's wrong. If he does nothing, he is open to charges of inattention, insensivity, incompetence, or perhaps discrimination. This could result in a lawsuit for damages caused by any physical, mental, or emotional disease which the student acquires in the next two years. They could surely be attributed to the teacher's neglect.

On the other hand, if he takes her aside, cleans her, and helps her change, then he is wide open to not only a lawsuit but even a criminal indictment for child molestation or sexual assault.

Two years ago, a school principal in Torrance was removed from office after three sixth grade girls accused him of sexual misconduct. They claimed that he had fondled them repeatedly in class. (Strangely, none of the 30 other students had ever noticed this.) When the case finally came to trial, the girls admitted that they didn't like him, and had made up the story "just to get him in trouble." After this, the school board restored him to duties as a teacher. They said they could not reinstate him as principal, because the parents had lost confidence in him.

Probable Solution:

Judges', whose professional competence seems more questionable than that of any other group, are protected from lawsuits by law concerning results of their actions even when they are proven to be wrong. There should be some sort of a buffer protecting a teacher from frivolous lawsuits.

There should also be a process to guarantee speedy resolution of the teacher's guilt or innocence. (Above all, we must avoid events like the McMartin circus. (After three years and three million dollars in county funds we are still six months away from even picking a jury. I know nothing about the defendant's status of guilt. However, after being out of work for three years, losing their homes, businesses, savings, and futures to attorney's fees, what real difference will it make if they are proven innocent? Will the prosecutor say, like the Linda Radnor character, "Oh -- I'm sorr--ree?"

SUMMARY OF PARTS I and II

Against all this practicality and worldliness, we want the teacher to make the right decisions, to reassert the value of truth and beauty, of learning for its own sake.

Let us remember though, that he is a teacher. He is neither God nor Devil. Neither hero nor villain, neither wizard nor charleton. He is a teacher. As such, he needs your support, both in finances, authority, respect, and trust. If you deny him these things, it is inevitable that he will fail. It is also inevitable that the cause of the failure belongs to society more than to him, so too will the cost of the failure belong more to society than to him. The choice is yours.

In section IC, we discussed some of the superficially troublesome aspects of mathematics, and how to eliminate them. Employing the philosophy of saving the best for last, let us now devote the rest of this paper to three final considerations of the nature of mathematics. However, these final aspects cannot be eliminated, for without them, mathematics no longer would be mathematics. Rather, the instructor should specialize in their explanation. Like caviar, truffles, or sweet breads, they are quite different. It takes time to develop a taste for them. However, once such a taste is acquired, nothing else will completely satisfy the appetite. So should it be for mathematics. With proper training it becomes indeed the epicurean delight of the intellect. This is the true test of both the teacher's scholasticism and petigogical skills.

PART III -- MATHEMATICS REVEALED
THREE INESCAPABLE ATTRIBUTES

A. Symbolism

Symbolism is a sort of code known by the mathematical community. Most graduates of any algebra course remember the word "of" invariably means times, which can be represented symbolically by a dot. They also recall that the bar of the fraction symbol means "divide by". However how many still get mixed up over the use of parenthesis - especially nested parenthesis? Moreover many never did fully understand the meaning of the symbol for cube root!

The higher one progresses in math, the more information is packed into each symbol. It takes several lectures to explain to a suitably trained student the exact meaning of the symbolism for the definite integral. Multiple integrals require even longer to have all the subleties of their symbolism explained. However such understanding is vital to mathematical competency. If the student has not learned his lessons, he is virtually helpless. The meaning of these symbols is almost impossible to guess, or figure out. Moreover, it is necessary that the student be aware of all of the implications and nuances of each symbol. Since different symbols convey different amounts of information a student may miss have the complete picture without suspecting that his understanding is deficient.

Many people incorrectly call this a math shorthand, or relate it to the chemical symbols. Both are improper analogies. Shorthand consists of strange looking symbols. However, when translated, these symbols stand for words that the student knows and understands. Each chemical symbol stands for a single element. This element may be unfamiliar to the student, but he has merely to learn its particular properties. Each symbol represents a lump of some kind of matter, and he knows what matter is.

With math the difference is major. The math symbol represents an idea that the student has probably never encountered before. He has to learn exactly what the idea represents. Moreover this idea may not be objects, but rather, concepts.

Proposed Solution:

This problem can be overcome rather easily by a careful instructor. Every new symbol should be defined so that the student knows what it means. The instructor would do well to define it again the next few days it is encountered. Some symbols require no more to be understood. Others however, will have to be explained carefully. Maybe a day or two after the explanation, the instructor will find it useful to facilitate learning through a few carefully selected counter examples.

There is no way to avoid this complexity. The instructor must recognize that some uses of symbolism will confuse his class, and have thought out plans to remove the confusion as soon as it occurs.

B. The Ultimate Question - Rigor

There are two properties that characterize math, that distinguish it from other subjects. These are rigor and abstractions.

The defining characteristic of mathematics is rigor. Without rigor, the subject studied is not math, but applications.

Rigor is probably best described as proof. It seems that other subjects teach that something is true because it is true; that is, some expert or group of specialists have agreed that it is true. However math leads its students to maturity faster. Nothing should be accepted on the basis of reputation. In mathematics "almost" and "usually" don't count. Until the student has proven a proposition himself, he cannot be sure of its universal validity.

The student is often surprised by this. Why must math prove all of it results? "In my other classes, we read the book, and accept what the others tell us. Why should math be different?"

There are two basic reasons for this:

1. Mathematics is an exact science. A statement that is true 999,999 times out of a million is a false statement. Mathematics tolerates no exceptions.
2. Many mathematics results are incapable of verification by experimental methods, the manner of "proof" so dear to other sciences. Hence, mathematical analysis followed by proof is the only way to guarantee accuracy. (What experiment could be performed to determine whether parallel lines actually do meet at infinity?)

Thus, since it is rigorous proof that is the essence of mathematics, and the bane of students, it is the teacher's responsibility to reinforce the former while overcoming the latter. He should welcome, not avoid opportunities to introduce proofs to the student, and vice-versa. I would recommend a gradual process, accelerating in both frequency and difficulty. Surely no course in mathematics should be finished without giving the student a few proofs, if for nothing else then to familiarize him with the process, if not the product. This is actually the truth and the beauty of mathematics! The instructor should consider it almost a matter of duty to guide students over this boundary which separates the purity and exactness of mathematics from the approximations of physical sciences, the generalizations and speculations of social sciences.

Most students have never seen a proof before. Those who have think of it as only a way to prove triangles congruent. They must be shown that the essence of proof is a chain of logical steps -- each of which can be justified by a reason which is known to be true -- leading from the original hypothesis to the final conclusion. Moreover most proofs use the language of algebra or analysis, not geometry. The instructor should train himself technically, and perhaps condition himself psychologically, so that imparting such knowledge is a pleasure, not a tribulation.

C. The Jewel in the Crown - Abstractness

Assume that a friend is describing to you an animal he has seen at the zoo yesterday. He keeps adding characteristics. However the more he adds, the more confused you become. Suddenly you recognize the animal and all becomes clear. You have enough information to recognize the animal. Even if you've never seen such an animal before, you can often assimilate it into a familiar pattern. A panda is just a friendly, black and white gorilla. However if the new animal is too different from your previous experience, you may have to build a new idea to accommodate such an animal. Many students have great trouble handling a rodent-like animal that lays eggs, or a duck that suckles its young. They have to construct a new category, the platypus. It takes a while to become familiar with the properties of such a strange creature. Intuition and common sense don't seem to apply.

Such is the case with most mathematical concepts. Often they correspond to nothing in the student's previous experience. Here, neither common sense or intuition apply.

The only properties such a concept can be assumed to have are just those properties which follow -- often laborously from the definition. These concepts are called abstract. They cannot be closely modeled by anything in the real world. They are defined as the totality of their properties. It is the nature of modern math that such concepts form its basis; derivatives, integrals (of various kinds), transform, essential singularity, moment-generating functions, etc.

These are difficult for the student to understand because they do not connect with anything in his previous experience. When he tries to model them with something familiar, he often gets burned (the case where the derivative is not the slope always appears on the calculus final). He eventually realizes that he must understand them on their terms, not his, a state which some students never reach.

This abstractness is probably the reason for most of the difficulty in higher math. Arithmetic rules, even if unknown, can be understood by trial and error. Although the student may not understand the rationale for long division, he can follow the receipe, and produce an answer which checks. Algebraic rules - we now use the word - theorems can also be verified by not too difficult trial and error. It often follows that just when the student becomes comfortable with some abstraction, it is generalized in another parameter. As if vector space is not abstract enough, it is soon itself abstracted into infinite dimensional vector space. While the professor is mumbling over equations he is scribbling on the board, all that enters the student's mind is, "I don't think we're in Kansas anymore, Toto." He's right, he's not!

The further the student goes in math, the less the rules of common sense seem to carry over. The first really hard concept in high school math was that of complex numbers, because nothing in the student's world had such properties. Therefore, he had to build a picture of it, by blending all these properties together, rather than by comparison to something known. Quite often, his blending left holes, or even worse, glossing over holes. In his desperation to create an acceptable model, the student equates the definite integral with the area under the curve. Since the average student would never think of a discontinuous integral, he would not realize that with such an occurrence, his model literally blows up.

To a large extent, higher math involves working with tools whose understanding takes some effort. Yet, we all know and accept that some properties of automobiles and microwave ovens do not follow common sense. (If the battery is more than 75% discharged, you can't recharge it by driving, you need to have it charged at a service station.) So we must accept the properties of our mathematical constructs, which obey higher rules than our terms of "common sense."

Now what can the teacher do? To help the student over the stumbling block of abstractness. He should:

1. Prepare him for its existence. "Calculus, while using ideas from algebra, geometry, and trigonometry, is none of the above." It involves telling him strange but powerful concepts that you've probably never seen or heard before. The first of these is limit. Now let's look at some properties of limits. Do not expect them to agree completing with all of your previous ideas. They are something new. Even if they appear strange at first, they are completely logical, and very useful.
2. Help the student build an approximate model of this mathematical beast.
3. After the student becomes comfortable with this model show him some of the differences between the model and the actual mathematical concept (if such exists).
4. Make him realize that he is now aboard the Starship Enterprise: "He is going where no man has gone before." None of the models connecting with his world will completely work for those nouns and verbs of higher math. They are new elements in a new world. However they are much more powerful, and much more useful than any tools he has ever used before.

   Although this is the complexity of math, it is also the strength, the power, even the beauty of math.

Hence, the student has become like the astronauts. He has left the familiar world, he is soaring through space and time exploring the new laws and the new worlds as he encounters them. For here is the true nature of mathematics. It is not the solution, but like happiness -- it is the pursuit. To explore new worlds!

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LADIES SECOND

A Study of the Perplexing Question of why Women are Consistently Outperformed by Men in Mathematics Tests

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OUTLINE

INTRODUCTION

At the turn of the century, at the request of their government, the French scientists, Binet and Simon, developed a test to measure the learning capacity of children with mental deficiencies. This led to the concepts of mental age, and I.Q. This test was modified in America by Terman at Stanford in the twenties. For some fifty years, this Stanford-Binet form of the IQ test ruled supreme. All of a person's mental ability could be revealed by merely giving him the test, and comparing his results with the sample used to calibrate the test.

However, in the troubled times of the sixties, various groups claimed that their low scores on such tests indicated not failure by themselves, but rather bias by the tests. Although this question is still being argued, it did popularize the idea that the tests may not be the objective panaceas that they had been considered. Eager researchers looked for deviations from the means of various population subsets. In the seventies, suspicion was cast on the validity of SAT scores in assessing the mathematical ability of women.

THE PROBLEM

Benbow and Stanley undertook an extensive study to test this conclusion. They chose as their population the group of gifted students who took the SAT test between 1980 and 1982. This group consisted of fifteen thousand boys and fifteen thousand girls. They found that for the verbal part of the test, the boys averaged 367 to the girls' 365. Essentially no difference. However, for the mathematics part of the test significant differences were noted. The boy's mean score 415 was significantly higher than the girl's mean of 386. In fact, this difference signifies that the boy's scores were 8% higher than the girl's scores. Statistically significant, but not overwhelming.

However, the more we study these results, the more staggering they are seen to be. First, boys scored 8% higher than girls overall. However, the higher we advance along the talent scale, the more overwhelming is male superiority.

Among the students who scored above 500, boys outnumbered girls 2 to 1. For those scoring more than 600, boys outnumbered girls 4 to 1. For the top group, those scoring more than 700 points, boys outnumbered girls a whopping 13 to 1.

More recent work substantiates these results (G. Child, Q. 31 U, 14, 1987). A study done by the Study of Mathematically Precocious Youth (SMPY), studied groups of students taking the SAT test in 1984 and 1985. It was found that 6% of the women received above 640, whereas the top 6% of the men scored over 700. Since many colleges use the 700 level as a cut-off for scholarships, men are 12 times as likely to qualify at these institutions.

Furthermore, this is not merely an American problem. In Jacobsen (1985), we find that in an international scholastic competition in Paris, 1983, there were 186 finalists from 32 different countries. Of these, seven were girls.

POSSIBLE ANSWERS

It has been found that in regard to mathematical abilities, boys and girls are about equal through age 12. Then, around the time when they transfer from junior to senior high school, the boys start outperforming the girls in math. As they grow older, this advantage not only persists, but increases. Therefore a logical question would be, "What begins as a student enters the teens, then increases?" Unfortunately, the answer is "Too much!"

Changes in Mathematics

First, math changes. Before, math was arithmetic. Now, it is algebra. Before it was concrete; now, it is abstract. Before it was manipulative; now it is analytic. Could response to some of these changes differ with the sexes?

Second, the students change. These children, or kids, undergo the transformation into adolescents, then adults. However, transformation seems to produce more changes in the female than in the male.

Physical Changes in Students

The physical changes become increasingly apparent as nature prepares their bodies for roles that nature planned for them a million generations ago. Besides these obvious changes, more subtle ones occur. The male's strength increases much more than the females, however her fine sense of touch and color perception greatly exceed that of the male. Could it not be that these changes are accompanied by invisible mental changes. After all, his testosterone is not reduced, whereas she is in a sea of androgen. As we can see, these are very active agents physically. Why should their action be confined to the physical nature of such biology. More about this later.

Chemical Changes in Students

Another difference is chemical. Up until now, the basic varieties of kids were pretty much the same. The chief differences were societal, not natural. However, they now diversify into chemical factories, making vastly different products. He is producing testosterone; she both progesterone and androgen. It would be difficult to find any materials more capable of producing change than these.

Psychological Changes (Attitudes)

There are also psychological changes. Kids react much the same. Boys and girls don't. Girls cry after seeing "Gone With the Wind;" boys don't (and kids, frankly, don't give a damn). Boys have been known to enlist after seeing pictures of Pearl Harbor; girls and kids don't. Girls have more auto accidents, but men have more serious accidents. Girls shoplift more, but boys burglarize more. No matter what the courts may say, boys and girls are different. Unemotional reflection will make clear that if boys and girls were not different, there would be no boys and girls.

Moreover, this difference has been going on almost since birth. Studies have shown that crying girl babies get attention from their mothers much sooner than crying boy babies. Girls get dolls; boys get balls or toy guns. Girls play alone, boys play together. Girls play inside; boys play outside. When they get older, girls play in the yard, but boys go around the block.

Psychological Changes (Goals)

Then, there is something which I call the "time factor." A woman will choose a major in order to have a specialty in which to obtain a job after graduation. She will need that job to support herself if single, or to have more things more quickly if married. However, the defining property, some say the raison d'etre, and the greatest joy of being a woman is to have children. When each is born, she will leave her job for a only a month or up to two years. Also, she will be away from her job when a child is seriously ill, has tonsils removed, breaks a leg, etc. Since her role of mother wil take so much time away from her job, why take all the extra time and effort to earn a degree in a difficult area like math. Let's choose something more in the real world (like Biology, or Psychology, or Business) or something closer to her mothering career (like teacher or nurse.)

In fact, it might be that the popularity of the above two careers among women might well be that they are an extension of the mother role into the workplace. Let us not miss the point that woman's lib merely means that a woman can obtain a male-role occupation if she wants to -- but she does not have to do so to make a point. (There are fewer females in the male dominated occupations of garbage collector, cattle "executioner," or armed robber.) Some women still prefer "women-type jobs.

Environmental Changes, (Personal)

Up until now, the girl views the other sex with distainful neglect. However, by the time she is in high school, this "boy" has changed to "guy," then "fella." Very often, he becomes "her fella" as such, no longer can be neglected. His opinion - right or wrong - counts. Often, she does things just to please him. Male opinions now are a force to be considered, whether they are those of a single male, or a group.

Historical Changes

All of the preceding changes are recurring. We expect that each generation of students will experience them. However, there also occured a series of changes of such profound magnitude and significance that they could occur only once. Our half of the century has been greatly shaped by these five occurrences. Each was enormous in its own affect, but each was further emphasized by the effect it produced on subsequent events:

1. World War II --
   Up until then women had been discouraged from entering the job market. However, now their country pleaded with them to take jobs to help win the war. The fact that almost 12 million men were gone, some for four years, made a job a welcome relief from boredom for many. This showed that women could do anything they had to do.
2. Women's Lib -- This showed that women could do anything they wanted to do.
3. Student Unrest (for want of a better word).
   The Great Rebelion of the Young against Authority and Convention -- This shaped and was shaped by campus unrest, the Political Protesters, the Hippies, Flower Children, the Free Speech Movement. This showed that women could want to do anything.
4. The Civil Rights Movement. Women are people too! They have rights -- maybe even quotas.
5. The Pill -- A woman's principal responsibility is to herself. She need not dedicate her life to the next generation unless she chooses to do so.

THE FUNDAMENTAL QUESTION

Math changes, and kids change. The concrete, manipulative, algorithmic, arithmetic becomes the abstract, logical analytic algebra. The student changes biochemically, emotionally, and psychologically in a personal environment which changes according to a personal maturation cycle, superimposed on the profound environmental changes which come about as the result of explosive rearrangements.

All of this confounds our attempts to answer the question, which of these many factors is the most significant to explain our question, why do women perform less favorably than men on math SAT tests?

SIGNIFICANCE

Most of our criteria for selection of students by colleges, for awarding them scholarships, for admission to specialized curricula, and for assigning them to different programs is based on SAT scores.

It is an established fact that women perform less favorably on the mathematics SAT test than do men.

Is this difference due to differing mathematical abilities? Are we unfairly penalizing the women? What can we do to bring up their mathematical performance? Should we change our methods of teaching. If so, how? Should girls be taught differently than boys?

THEORETICAL BACKGROUND

Learning

Learning, as a form of behavior, is fundamentally a biological process made possible by the interaction of neuroanatomical structures and the network of neurons comprising the Central Nervous System (CNS) (Cotman and McGaugh, 1980; Gesell, 1945; Piaget, 1971). Although the capacity for precocious reasoning is certainly nurtured by environmental factors (Gesell, 1945; Piaget, 1979), it is initially a function of the genotype, and the structures and systems that enable learning to unfold in the developing fetus in a carefully controlled and regulated manner (Berrill & Karp, 1976; Piaget, 1971). Piaget (1970, p. 18-19) commented: "In searching for the beginnings of logical structures we immediately get into the realm of the coordinations within the nervous system and the neuron network."

Logical thought or the ability to reason, as a correlate of learning, originates in the DNA blueprint that guides development. This is not to suggest that learning and precocious reasoning are solely genetic derivitives, however. As Berrill & Karp (1976) pointed out, it is likely that the environment exerts a considerable shaping force on the ontogeny of behavior.

This notion of gene-environment interaction in the expression of cognition is widely supported in the literature and helps refine further a definition of learning (Parsons, 1980; Petit, 1972; Piaget, 1971). According to piaget (1971, p. 21): "The epigenesis of the cognitive functions, like any other, does, in fact, presuppose an increasingly close collaboration between the factors of the environment and the genome, the former increasing in importance the larger the subject grows." Petit (1972, p. 42) continued: "When it comes to man, a large environmental variability is superimposed on the considerable genetic polymorphism in the biological substrate of behavior and intellect." In summary, learning evolves from a genetic substrate and is shaped and refined by environmental forces. Additionally, learning can be defined on two levels; as a capability or a potential at the genetic level and as a behavior at the environmental level.

Personality theorists concur with the biologists in this assessment of learning as a product of both nature and nurture. Miller & Dollard (1941, p. 1) wrote: "To understand thoroughly any item behavior...one must know the psychological principles involved in its learning and the social conditions under which this learning took place. It is not enough to know either conditions or principles of learning; in order to predict behavior, both must be known." The behavior of learning is shaped by modeling and operant conditioning as is all behavior (Bandura & Walters, 1963). Further, classroom and learning behaviors are continually shaped by the changing needs of the emerging personalities (Bandura, 1977; Bandura & Walters, 1963).

Murray (1938) sought to unravel the complex relationship between personality and achievement by speculating on how the environment shapes the structures of thought. He postulated the existence of various "directional forces within the subject which seek out or respond to various objects or situations within the environment (Murray, 1938, p. 24). These needs, and there are many categories of them, are viewed in terms of the type of effect they have on the individual; the response elicited from him or her. Murray (1938, p. 40-41) termed these effects "presses" and said: "a press is a temporal gestalt of stimuli which usually appears in the guise of a threat of harm or promise of benefit to the organism." One category of needs particularly germane to the present study are the socio-relational needs. Murray (1938, p. 83) theorized the existence of a need for affiliation and defined it as "a need to form friendships and associations. To greet, join and live with others. To cooperate and converse sociably with others. To love. To join groups." In a similar vein, Abraham Maslow (1970), viewed the human personality as a dynamic seeker of need satisfaction and included belongingness, self-esteem and self-actualization in his needs hierarchy. This need to realize one's full potential is particularly pertinent to the study of the gifted, encouraging the idea that settling for merely average when one is truly brilliant is not only a waste of a natural resource but also a thwarting of normal personality development.

In summary, as genetic considerations guide the development of personality, so the personality shapes the behavioral repertoire, being formed of a pattern of environmental perceptions and need responses. The capability for learning underlies the development of personality but learning as a behavior is also a pattern of classroom specific responses and possibly a need which is a driving force for self-actualization. Even more pertinent to the present study is a discussion of complex reasoning ability, arguably the most advanced type of learning.

Reasoning Ability

Corollary to the process of learning is the development of rational thought (Cotman & McGaugh, 1980; Piaget, 1970, 1971). Whether termed logic (McGonigle & chalmers, 1977), analogical reasoning (Gillan, Premack & Woodruff, 1981), metacognition (Shimp, 1982), or formal operational thinking (Piaget 1950, 1970, 1971), when an individual develops this pattern of thought he/she can view a problem with objectivity instead of with the complete subjectivity characteristic of previous stages of cognition (Piaget, 1958). Several theorists believe that maturation is the key to the development of reason (Gesell, 1945; Piaget, 1958; Solso, 1979). Solso (1979, p. 347) wrote: "The critical feature of this viewpoint is that thought develops -- that is, that thinking activity changes in an orderly fashion as individuals progress from healthy infants to mature adults." Epstein's (1974a 1974b) studies measuring mind and brain growth spurts lend support to this developmental analysis by suggesting that there are indeed cȩrtain ages when healthy individuals experience accelerated growth in their mental capabilities and in their brain and skull size. Piaget (1950, 1970) postulated the progression of thought from a preoperational stage, through a concrete operational stage, culminating in a formal operational stage, at which point analytical abstraction or complex reasoning becomes possible.

Existing evidence, then, indicates that all children, regardless of intellectual ability, follow the same general pattern of stage development. In an effort to link cognitive and chronological development, Piaget (1950, 1970, 1971) searched for age markers indicative of the various stages of thought. Specifically regarding the development of logic and formal operational thought, Piaget (1958, p. 43) commented: "from ages 7-8 until 11-12, there is a consistent effort on the part of thought to become more and more conscious of itself (and by) ages 11-12, thinking is a logical experiment which comes as the completion of mental experiment." In summary, it seems that early adolescence is a critical age in the development of reasoning ability.

The research investigating the relationship among age, reasoning ability, and the Piagetian stage of formal operations is relatively scarce. The majority of the existing studies attempt to correlate scores on Piagetian tasks with measures of IQ (Brekke, Johnson, Williams & Morrison, 1976; Brown, 1973; DeVries, 1973; Kuhn, 1976; Little, 1972; Radar, 1975). Since reasoning ability is only one component of one component of intelligence, those who assert its presence from correlations between IQ scores and scores on Piagetian tasks are making a conceptual error. A more appropriate correlation would be between scores on a measure of reasoning ability and scores of Piagetian tasks. Two studies were located that investigated such a relationship.

Keating (1975), studied 100 fifth and 100 seventh grade boys classified as bright or average on the basis of their percentile ranking on the Iowa Test of Basic Skills. The final treatment groups contained 50 students from each age and ability level. These students then took the Raven's (1960) Standard Progessive matrices and completed four Piagetian tasks: one test of concrete operational skills and three tests of formal operational abilities. Keating (1975, p. 279) confirmed the hypothesis "that brightness as measured by a psychometric test of abstract reasoning ability implies developmental precocity in reasoning." The generalizability of Keating's findings are limited, however, since all of his subjects were males.

Shayer, Kuchermann & Wylam (1976) investigated not only the attainment of formal operational thought but also progression through this critical stage for students of various ages and ability levels. This study made a significant contribution to the literature on the development of precocious reasoning ability for several reasons. First, the authors used a test of reasoning ability, not an IQ test, to determine the students ability levels. Second, a gifted treatment group was identified for the data analyses. Third, formal operational thought was investigated on two levels, early and late. Fourth, 12, 13 and 14 year olds were identified. This allowed for a greater understanding of the precise development of formal operations at each age level for both talented and more typical students. Fifth, 10,000 subjects were studied, 800 of whom were identified as gifted. This large subject pool elicits a high degree of confidence in the results and conclusions of this study.

The 10,000 participants were given the Calvert Non-Verbal Test of Reasoning to determine their ability levels. The subjects were subsequently classified as high ability, average ability and low ability and further classified within three age groups: 12 years 2-4 months, 13 years 2-3 months and 14 years 2-3 months. Cognitive development was inferred from scores on the three Class Tasks (Piaget, 1971).

Task I was drawn from The Child's Conception of Space (Piaget & Inhelder, 1956). Task II was drawn from The Child's Construction of Quantities (Piaget & Inhelder, 1974). Task III was drawn from The Growth of Logical Thinking (Inhelder & Piaget, 1958). The questions that comprise Task I measure the development of thought from the pre-operational stage through the late concerete operational stage. The items of Task II assess cognition as it progresses from early concrete operational forms through the early formal operational stage and the elements of Task III test the development of reason, tracking thought as it advances from late concrete operations to late formal operations. Most relevant to this study are the results from Tasks II and III.

For Task II, at every age level, a higher percentage of the high ability group, as compared with the average and low ability groups, used early formal operational thought to solve the problem. Indeed, at ages 13 and 14 nearly 90% of the high ability students used complex thought compared with slightly more than half of the average ability subjects.

For Task III an equal number of high and average ability students used early formal operational thought at ages 12 and 13, and at age 14, a larger percentage of average students, as compared with the high ability group, were at this cognitive level. This apparent confusion is explained by examining the relative percentages of students who used late or advanced formal operational thought in completing Task III. At every age level the percentages of subjects who used advanced complex reasoning in solving this task far outpaces the percentages of average ability subjects judged to be at this cognitive level. For example, at ages 13 and 14, 46% and 58%, respectively, of the top ability youngsters had progressed to advanced reasoning levels, compared with 13% and 23% of the average ability students. The data from this study corroborate the idea that reasoning ability is linked to Piagetian development and that high ability students progress through the stage of formal operations more quickly than their less able age peers...In summary, it seems that cognitive differences between these two intellectually diverse groups are found in the the rate and quality of their learning (Benbow & Stanley, 1982b; Carter & Ormrod, 1982; Keating, 1975; Shayer, Kuchermann & Wylam, 1976). It also appears that the development of formal operational thought at approximately ages 10-13 is consonant with the development of logic. Further, there is a link between a rapid progression through this Piagetian stage and the development of exceptional reasoning ability, but much more research is needed in this area (Keating, 1975; Vandenberg, 1972). One reason for studying reasoning ability is to better understand the development of mathematical precocity and the learning of mathematics in general.

Mathematical Reasoning Ability

The importance of mathematics in the development of civilization can hardly be underestimated. From the times of the Egyptians and the Babylonians as far into the future as can be predicted, mathematics provides the foundation for inquiry into philosophy, technology and education (Eves, 1976; Sahakian, 1968). Alarming statistics suggest that the quality of education in the United States is declining, particularly in mathematics (Lerner, 1982). This possibility lends urgency to the issue of the math achievement of the country's most talented students. If the U.S. is to continue its technological prominence in the world, this capacity for exceptional performance in mathematics must be discovered and nurtured to its full potential. Additionally, there are two important reasons for focusing on this capacity as it develops in adolescence. First, as previously noted, ages 10-13 is the approximate age period of the onset of formal operational thought, therefore the time at which understanding the logic of higher mathematics such as algebra, geometry and calculus becomes possible. Second, there is a need to understand the development of precocious reasoning ability and its relationship with superior achievement in mathematics to better aid these gifted youngsters in their emotional and academic development.

As noted by Marland Jr. (1972, p. 2), gifted youngsters are those "who by virtue of outstanding abilities are capable of high performance." Julian Stanley, founder and director or the Study of Mathematically Precocious Youth (SMPY) at the Johns Hopkins University, has focused his research specifically on youngsters who are capable of outstanding performance in mathematics. When asked about his reasons for concentrating on this population, Stanley (1977a, p. 77) commented: "It was necessary to choose school subjects much more highly dependent for their mastery on manifest intellectual talent than on chronological age." In the same paper, Stanley mentioned many benefits both to the individual and to society of cultivating mathematical talent.

Some of them are:

1. That the student will experience enhanced feelings of self-worth and accomplishment.

2. That the student will be better prepared for early entrance into college, allowing more years for career and creative pursuits.

3. That society will gain scholars who are ably prepared to major in mathematics and in related fields.

4. That society will gain happier, more productive citizens, who are realizing their full potential and who know how to educate their own children.

5. That society will bear less educational costs, as these students typically spend far less time in the traditional 16 year public educational system.

Finally, with regard to the reasons for focusing on the mathematical talents of precocious 10-13 year olds, it is important that the identification for mathematical talent is relatively straightforward (Fox, 1981a; Stanley, 1977a). The mathematics section of the Scholastic Aptitude Test (SAT-M) is a psychometrically sound instrument that has been normed on gifted populations and has proven useful in the discovery of exceptional reasoning ability (Cohn, 1980; Fox, 1981a; Stanley, 1977a). An interesting and persistent finding concerning the SAT-M scores of math able students is the disparity in performance between males and females, which consistently favors the male.

Gender Differences in Mathematical Reasoning Ability

In 1980, the magnitude and implications of the observed disparity between male and female performance was brought to the forefront in an article in Science magazine written by Camilla Benbow and Julian Stanley. The conclusions that these authors reach about the causes of the observed gender differences in performance are eclipsed by the sheer power of their raw data. In six years of testing approximately 10,000 seventh and eighth graders, Benbow and Stanley (1980) documented consistent differences of 30 or more points between the girls' scores and the boys' scores on a measure of math reasoning ability. Further, the authors found the greatest disparity at the upper, above 600, ranges of the scale (Benbow & Stanley, 1980). In the 700-800 range, which represents the 95th percentile of college bound high school seniors, the seventh and eighth grade gifted boys outscored the same aged gifted girls by a ratio of 4:1 (Benbow & Stanley, 1980; Fox & Cohn, 1980). Also, for each year that data were collected the percentages of males and of females scoring at or above 600 were calculated. In four of these six years the percentage of girls reaching or exceeding 600 did not reach even 1% of the girls tested, while the percentage of boys reaching or exceeding 600 during the same four years averaged nearly 18% (Benbow & Stanley, 1980, 1982a, 1982b). The largest difference recorded was in 1976 when 58.3% of the eighth grade boys tested scored above 600, compared with not one eighth grade girl (Benbow & Stanley, 1980, 1982a, 1982b).

THE SOLUTION

Because of the relatively brief time that this problem has been actively studied, and especially because of the extremely high level of reader predilection in this area, I will present three different possible explanations.

Version I: Continuing Environmental Pressure Recommended for Female Audiences

As the case for all other living organisms, humans produce babies. However, they remain merely "babies" for only a short time. They soon evolve into "baby girls" and "baby boys." They are even color-coded in pink and blue, for easy identification! From then on, they are treated unequally. Parents, relatives, friends -- society in general -- impress different preconceived patterns on them. Naturally, they develop differently. Some talents are reinforced; others are extinguished. Girls play with assembled toys like baby buggies. Boys play with creative toys like Lincoln Logs and Erector sets. Because of their physical and sexual vulnerability, girls are kept on shorter reins. When girls play alone (dolls); boys play in small groups (cowboys). By the time girls play in small groups (house); boys play in large groups (ball). Small girls play in the house; small boys play in the yard.

By the time girls are considered old enough to play in the yard, boys are allowed to go around the block. By the girls are allowed to go around the block, boys have gone to the playground. When they finally get to the playground, girls play cooperatively (swings); boys play competitively (baseball). When girls compete, they do it in non-contact sports (tennis). When boys compete they do so in contact sports (football).

To really understand the difference watch two "single-sex" basketball games; girls throw the ball back and forth, dribble resolutely, throw the ball into the hoop, and wait to catch it when they miss. Boys charge through defenses, force shots, and "crash the boards," when they miss. In short, girls play finesse basketball; boys play "alleyball." This typifies the roles that society has forced on its children. Girls do what they're told; boys stick up for their rights. Girls "act like ladies;" boys "don't let them push you around."

All through childhood, girls cooperate; boys compete. Girls are cheerleaders; boys are linebackers. Girls are part of a group (song-girls); boys are leaders (quarterbacks). Girls watch; boys do.

The same differential treatment carries over into work. Girls are told, "That's too hard for you to do, leave it for your brother." Girls wash the dishes; boys fix the fence. Girls wait on customers; boys unload trucks -- or drive them. Girls do patterned thinking (bake a cake); boys use abstract thinking in their jobs, ("fix" the TV).

As they grow up girls are fed the same myths: Girls have to be protected. Girls shouldn't be competitive. Girls shouldn't do hard things. Girls shouldn't explore new things. Girls shouldn't deal with abstract ideas. In fact, girls shouldn't work with ideas at all. They should work with people. And above all girls must be polite; they should do what they are told to do.

All through their impressionable years, girls are bombarded by the same messages. Although it may be false; it is continual. Perhaps the worst part of all is that the girls are never given this information of inferiority directly. If such were the case, they would perceive it as false and challenge it. Instead, for the manner of their treatment they gradually deduce what the messages are. Another unfortunate aspect of all this is that the smartest girls deduce them first. They are most affected by them when they are least likely to question them. Since girls are supposed to be polite, not competitive, to do what they're told, they do not challenge such nonsense.

Shortly before they enter their teens, many things change. Among the chief of these is their attitude toward the opposite sex. Their feelings toward boys gradually shift; ostracism, avoidance, tolerance; acceptance, interest, desire.

Since society has decreed that girls are passive, whereas boys are active; she must wait for him to ask her for a date. Suddenly, what he thinks about her is important. Even if she has no romantic interest, she has to have a date for Saturday night because now all of her peers do. Suddenly, there are very few activities available for unpaired females.

How does all this affect the girl in the early teens? To find out, let's first see how it affects boys. Boys compete in everything; against everyone. They don't like losing -- in anything. Moreover, it is especially humiliating to be excelled by -- a girl! Boys experience such feelings especially during this age period. Because of this, the smartest girls often have the fewest dates. However being smart, these girls figure out the problem rather quickly. "If it means so much to him to do better than I do in class, I'll let him." Not only is this good strategy to be "popular," but I'm not compromising my values, since I was never taught to compete!"

The need for the boy to excell over the girl is most pronounced in math. Not only must the boy always compete, he should always win. Moreover, the harder the quest the greater the victory. Therefore, since math is the "toughest" subject, it is here that he must win!

As a supposition of her role of mother, society has extended motherly characteristics to its females. The preceived role of the female was to serve, to help, to "make things better." Hence, the girl wants the boy to get the highest grade in the class because it will make him feel good.

Such a combination of pressures causes the girl to not do as well in math tests as boys. Then when she starts to be outperformed, her parents, counselors, teachers, feel that their similarly acquired feelings of feminine inferiority have been proven. Hence she is increasingly steered away from mathematics. Thus the prophecy has become self-fulfilling; women don't do as well in math as men!

Version II: Sudden Irreversible Biochemical changes: Recommended for Male Audiences.

The reasons for these observed differences in performance are difficult to delineate. It is well-known that sex differences exist in human behavior (Maccoby & Jacklin, 1974; Parsons, 1980). It is equally obvious that there are differences in human anatomy and physiology, and it is reasonable to speculate that these differences are based on the variance between the male genotype (XY) and the female genotype (XX). Given this, it is logical to assume that there are gender differences in Central Nervous System (CNS) organization and in neuroanatomy.

The original support for this hypothesis is found in the literature on the sex hormones: estrogen, progesterone and, most critically, testosterone, Levine (1966), after noting the extensive research documenting sexual dimorphism in the mammalian brain, commented: "According to this evidence there are distinct differences between the male and the female brain in a mammal" (p. 84). A plethora of studies (Breedlove & Arnold, 1981; Conel, 1963; Cotman & McGaught, 1980; Dorner & Staudt, 1969; Harris & Levine, 1965; Landsdell, 1964; Levine, 1966; MacLusky & Naftolin, 1981; Raismann & Field, 1973), support the contention that there are reliable sex differences in mammalian CNS organization and brain structure. Overwhelmingly, this research implicates the sex hormones as the primary causal agents in the development of these differences. Indeed, it is thought by some researchers that the fetal androgens regulate the differentiation of the brain into a male type or into a female type, depending on the amount and type of androgen (Harris & Levine, 1965; Konner, 1982; Levine, 1966). Although the vast majority of the evidence for the existence of sex differences in the mammalian brain comes from animal studies, it is nonetheless reasonable to assume that these differences are also present in the human brain (Kandel & Schwartz, 1982; Levy, 1981). Jerre Levy (1981, p. 209) concluded: "It would be surprising (if) the sex hormones affected neural organization and behavior in other mammals, but not in people."

As reported by Levy, the evidence for human gender differences in learning behavior suggests that boys and girls employ different problem solving strategies and that the choice is, at least partly, a reflection of hemispheric activity in the brain

(Cioffi & Kandel, 1979; Beckman, 1978). Further, Levy (1982, p. 211), noting the evidence that males surpass females in mathematical reasoning ability (Droerge, 1967; Hilton & Berglund, 1971; Svensson, 1971; Very, 1967), suggested that this male superiority is probably causally related to a "male maturational advantage in particular subsets of left hemisphere functions." She then presented a plausible scheme for sexual dimorphism in brain lateralization, postulating that the hemispheres of the female brain become "organized for communication (and) context sensitivity or field dependency" (p. 213). On the other hand, the hemispheres of the male brain "would become organized for highly structured schemata" (p. 212) and more abstracted representations. Other researchers also suggest sex-related differences in brain hemisphere lateralization (Guy & McEwen, 1980; Reid, 1980; Wittig & Petersen, 1979). Hypotheticaly, this variance in brain organization affects the development of cognition, fostering the gender differences observed in mathematical reasoning ability.

This result is probably so unanticipated that it would require substantiation. It has been shown (Kolata 1983), that these male sex hormones should react with other biochemical substances present in the most mathematically gifted males to produce higher than average occurrences of left-handedness, allergies, and near-sightedness.

Benbow and Stanley (1983) found in their sample of 34,820 mentally gifted students that among the males: 20% were left-handed, 55% had allergies, 70% had myopia.

While surely not a proof, these data should furnish a strong corroboration of the theory that strong mathematical ability is related to the level of male sex hormones. In this, males clearly have an advantage.

SUMMARY OF VERSION II

1. It is known that the right hemisphere of the brain controls mathematical reasoning.
2. Norman Geshwind of Harvard Medical School postulates that excess testosterone causes the right side of the brain to develop more than usual (Marx, 1982).
3. Testosterone is the male hormone.
4. Therefore, males will have more mathematical ability than females (Kolata, 1983).
5. It is known that the brain controls "cross-over."
6. Therefore, the right hand side of the brain controls the left hand side of the body.
7. Therefore, mathematically gifted males should be left-handed more than the usual.
8. Too much testosterone can cause immune system disorders (such as allergies and asthma).
9. Too much testosterone can cause myopia (nearsightedness), (Geshwind, 1982).
10. Therefore, mathematically talented males should have larger than average amounts of left-handedness, allergies, and nearsightedness (Annett, 1985).
11. All three of these results have been observed in mathematically talented males (Benbow and Stanley, 1983). Left-handedness is twice average; allergies, five times average. Moreover, 70% of these mathematically talented males were near-sighted.

(For what it is worth: Your author, a male professor of mathematics, is very near-sighted, left-handed, and possesses dozens of allergies!)

Because of its Importance, We Introduce This Paper as

"EXHIBIT A"

Math Genius May Have Hormonal Basis

During the past several years, Norman Geschwind, a neurologist at Harvard Medical School, has proposed that left-handedness and immune system disorders might occur together and that they will frequently be linked either to serious abnormalities such as autism, dyslexia, or stuttering or to certain kinds of giftedness, particularly artistic, musical, or mathematical talent (*Science*, 9 July 1982, p. 141). "There's been—understandably—an enormous degree of skepticism," says Geschwind, but his idea has also stimulated some scientists to look again at their own data.

The most recent researchers to look again are Camilla Benbow and Julian Stanley of Johns Hopkins University who study mathematically precocious youth. To their surprise and delight, they find that Geschwind's predictions hold up beautifully in their group. Moreover, they believe that Geschwind's proposal might explain why the most mathematically gifted students are almost entirely male.

Geschwind proposes that excess testosterone or unusual sensitivity to testosterone during fetal life can alter brain anatomy so that the right hemisphere of the brain becomes dominant for language-related abilities and the person is left-handed. The association with the immune system arises, Geschwind suggests, because testosterone production, sensitivity to testosterone, and the activity of the immune system are genetically linked.

The link with mathematical genius occurs because mathematical ability is generally thought to be a right brain function. "If you get the mechanism adjusted just right you get superior right hemisphere talents, such as artistic, musical, or mathematical talent. But the mechanism is a bit treacherous. If you overdo it, you're going to get into trouble," Geschwind says. "It's a funny mechanism. At first, it looks like you have to deliberately produce damage to produce giftedness."

When Benbow and Stanley at Johns Hopkins learned of Geschwind's hypothesis they were intrigued. They had data from nationwide talent searches for mathematically gifted seventh graders (*Science*, 2 December, p. 1031). To find these students, they looked at scores on the mathematics section of the Scholastic Aptitude Test, a test designed for 11th and 12th graders. The very best students are those who score above 700. Benbow and Stanley estimate that these seventh graders are the top one in 10,000 in their age group. They decided to contact these students to see if they are left-handed and have immune system disorders.

Twenty percent of these mathematically talented students, Benbow reports, are left-handed, making them more than twice as likely to be left-handed than the general population. Sixty percent of them have immune system disorders, which is five times the incidence in the general population. These disorders, Benbow says, are generally "symptomatic atopic disease," better known as allergies and asthma. They also asked about myopia and learned that 70 percent of the high scorers are nearsighted. (Geschwind says that there is a correlation between intelligence and myopia, which he is now investigating.)

When the Hopkins researchers moved down the list of high scorers to students who were not so gifted, they found that the students were less likely to be left-handed, have immune disorders, or to be myopic. When they got down to the students who scored not much better than chance on the SAT math test, they found that the incidence of these conditions is about the same as those in the general population.

If testosterone during fetal life does all that Geschwind believes it does, it might be expected that boys, who are exposed to more testosterone in utero, would be more likely than girls to be affected. Males are more likely than females to be left-handed, to have immune system disorders, to stutter, to be dyslectic, to have autism, and, according to Benbow and Stanley's work, to have high scores on the math portion of the SAT. Among the nearly 50,000 seventh graders who took the test, they found 260 boys but only 20 girls who scored over 700—a ratio of 13 to 1. But in a similar search for verbally talented youth, there were equal numbers of boys and girls among the high scorers. Once again, Geschwind is not surprised, saying that his theories do not provide "a mechanism for giftedness in verbal areas."

But if Geschwind is correct in his predictions and if the Johns Hopkins group really is detecting inborn mathematical precociousness, boys are going to be a very variable group. They can be geniuses or they can have severe learning problems. "I think that if you look at the group of people who are very bad in math there will be an excess of males there too," says Geschwind. But the data so far on the precocious students, he remarks, "Fit in perfectly, to put it bluntly."—GINA KOLATA

Version III: A Bridge Over Troubled Waters. A Joining of These Two Previous Views. For Use With Mixed Audiences

Perhaps both these previous versions are partially correct. Surely boys are different from girls! The causes for these differences are both the environmental pressures of society, and the biochemical changes produced by the sex hormones. Let us now consider an approach combining these views.

In spite of the growing body of biological research linking the fetal androgens to gender differences in reasoning ability, the nature versus nurture controversy rages on. Fennema and Sherman (1977) supported the nurture side of the argument, providing data that gender differences in mathematics achievement are primarily the result of differential exposure to mathematics. Fennema (1980), reviewed evidence from several longitudinal mathematics enhancement projects, considered affective and educational variables, and concluded; "Certainly when both females and males study the same amount of mathematics, differences in learning mathematics are minimal and perhaps decreasing (p. 90). She continued: "Far fewer females elect to study mathematics (because of) females' lesser confidence in learning mathematics and a belief that mathematics is not useful to them" (p. 90). Benbow and Stanley (1980, 1982a, 1982b) disagree.

In the now famous 1980 article in Science magazine, Benbow & Stanley (1980), published data that indicated the existence of persistent gender differences in math reasoning ability among young, very bright students. The fact that these youngsters were mostly seventh and eighth graders is important because these grades are far before the years when boys and girls participate differentially in mathematics, yet at these ages they vary dramatically in their performance on a measure of mathematical reasoning ability (Benbow & Stanley, 1980, 1982b). Further, follow-up studies have indicated that as these students mature, the boys' performance improves more than does the girls' all of the way through senior high school, even when the students enroll in the same courses (Benbow & Stanley, 1982b). These authors suggest the opposite scenario from Fennema and Sherman: that ability differences among boys and girls foster the attitudinal differences among them that foster differential college course taking (Benbow & Stanley, 1980, 1982b). Finally, it must be remembered that Fennema and Sherman talked about differences in mathematics achievement, while Benbow and Stanley discussed gender differences in mathematical reasoning ability. The relationship between these two variables is still not clear.

In their data on reported attitudes toward mathematics, Benbow and Stanley (1982b, p. 617) concluded:

1. Boys and girls equally say that they like mathematics.
2. Attitudes toward mathematics could not predict the number of high school mathematics courses taken or SAT-M scores, or scores on the College Board's Math Level I Achievement Test.
3. Girls probably participate less then boys do in mathematics, not because they like it less, but because they prefer verbal areas. The relevant aspect of this endless controversy is not why girls and boys differ in mathematical reasoning ability, but that they do differ, and that something must be done about this variance.

Returning to the biochemical level, (Moos and Kiritz, 1974:100) found that competitive and environmental pressure to perform resulted in a state of physiological arousal involving biochemical compounds similar to those implicated in learning (Kandel & Schwartz, 1982; Kiritz & Moos, 1974; Rose, 1982).

In another study, Johnson, Johnson and Anderson (1978) investigated the effects of competitiveness on attitudes toward learning and concluded that, at the junior high and senior high levels, competitiveness was positively related to getting good grades. Further, these authors found that cooperativeness, defined as a desire to work with, and to be close to others, was positively related to being liked by others, and to enjoying doing homework together in small groups, (Johnson, Johnson & Anderson, 1978). Researchers at the Johns Hopkins University have gathered evidence indicating that competitiveness is related to the investigative approach that typifies the male preference in learning style, while the attributes of cooperativeness typify the female approach (Fox, 1976).

Studies were conducted by a Harvard University research team that evaluated the Harvard Project Physics Course. This was an advanced math and science course taken by 2500 tenth, eleventh, and twelfth graders in more than one hundred classrooms throughout the United States and Canada during the 1967-1968 school year. Using these data to evaluate student perceptions of the classroom environment, Walberg (1969a, 1969b), found that perceptions of social climate, including intimacy and competitiveness, did indeed predict achievement for students enrolled in the Project Physics Course. Additionally, the prediction remained significant after IQ and initial ability level were compensated for (Walberg, 1969a, 1969b, Walberg & Ahlgren, 1970). Studying a random sample of 800 of these Project Physics students, Anderson (1970) concurred that certain aspects of the classroom do indeed affect learning and achievement, and that the majority of the variance in performance was attributable to the gender and to the ability levels of the participants.

Other researchers support this contention (Fox, 1976; Fox & Cohn, 1980; George & Denham, 1976). For example, interest inventories administered to Study of Mathematically Precocious Youth participants indicated that the girls were most strongly oriented toward social activities, while the boys preferred more investigative and independent tasks (George & Denham, 1976). Capitalizing on this finding, Fox (1976) attempted to describe its relationship to mathematics achievement by creating an all-female class for mathematically precocious girls. The particular contextual components of this class were a strong emphasis on social aspects, including: small group intimacy, classroom involvement and an emphasis on cooperation as opposed to competition (Fox, 1976). She concluded that when the girls perceived the classroom as socially affiliating and involving, their achievement in mathematics improved.

Finally, with regard to gender differences, personality variables, and math achievement, Chansky (1966), administered the Children's Manifest Anxiety Scale, the Primary Mental Abilities Test and a teacher-made algebra test to ninth grade algebra students. He concluded that, for the girls, anxiety was negatively related to achievement in algebra. That is, the greater their anxiety, the lower their grades. For the boys, he found no such relationship (Chansky, 1966). It seems then, that in the traditional classroom, anxiety may be generated by a competitive atmosphere, and may inhibit the learning and the mathematics achievement of girls.

In summary, it thus seems that whether students consider the classroom as either cooperative or competitive in nature will have an impact on their achievement. Further, it is likely that boys and girls prefer different types of mathematics classrooms, and boys preferring an independent, investigative mode, and the girls favoring a more social, interactive approach. Finally, it is possible that boys and girls attune themselves to different social climate dimensions within the classroom, in accordance with their varying preferences and that these perceptions may be reflected in their achievment.

SUMMARY OF THE COMBINED VIEWPOINT

Thus, it would seem that at least some of the reason for the lower math scores may be caused by society's policies:

1. Boys are encouraged to compete; girls aren't.
2. Boys become so used to competition that they do not react negatively to it. Girls, due to the lack of experience with competition perform less well in a competitive environment.
3. Boys, therefore prefer a competitive environment and perform best in such. The opposite is true of girls.
4. Because SAT tests are administered in an extremely competitive environment, male achievement is promoted; whereas female achievement is impeded.

Once again, we see that society's higher regard for male performance, coupled with lower expectations for female achievement is self-fulfilling in all areas.

Moreover, since mathematics is popularly viewed as the most difficult subject, male superiority should be most pronounced here. This bias is also self-fulfilling.

HORRIBLE HUNCH

We have seen that:

1. By socio-environmental pressures, we have conditioned males to respond better in competition, whereas females perform less favorably under such an environment.
2. Our classrooms are structured to that there is much competitive pressure.

Could it possibly be that in school girls actually learn less than boys because of the environment we have created?

OVERALL SUMMARY

It is apparent that significant gender differences exist in mathematical reasoning ability. Further, it is likely that these differences are functionally related to sexual dimorphism in brain organization, particularly in the lateralization of the brain hemispheres. However it is equally likely that such differences are caused by society forcing the female into a role of "actively accepting" male dominance in academics in general, but math in particular.

Probably the true solution consists of blending both of these approaches together, as indicated in the third alternative solution.

While biological researchers continue to gather the physiological and neurological evidence, the behavioral scientist strives to explain reasoning ability in a different way: from an environmental perspective. Surely the controversy will continue.

IMPLICATIONS FOR HIGHER EDUCATION

1. Admission and Financial Aid

It is known that men outperform women on the SAT math tests. However, the purpose of these tests is to identify the mathematically gifted. However there is more than some uncertainty that this differential in scores is caused by an actual difference in math ability. If the cause of this difference is glandular secretions, or environmental practices, then it really should not be used as an assessment of mathematical ability. However, it is so used for purposes of both admission policy and financial aid in the form of scholarships. Moreover, these SAT scores seem to be most used by the most prestigious schools. Therefore, the probabilistic rules of sex determination at the moment of conception determine both admission and financial aid. They, more than motivation, effort, and talent decide on the mix of the entering college class. If such causative factors influence SAT tests scores, which in turn influence what colleges are available to female students, then such use of these scores constitutes hidden discrimination.

Some mathematical formulas should be worked out to overcome the effects of such unconscious bias. That is, it should be possible to weigh such scores so that two people of the same intelligence should receive the same score on the same test regardless of genetic history. This will remedy some results of this genetic bias.

2. Advisement

The population in general, and counselors in particular should abandon the idea that certain occupations are male or female. Diplomas come bound in leather folders. They wear neither skirls nor pants. This goal seems already in sight. A generation ago, society got over the shock of the female doctor (physican). Today, it seems almost willing to accept the male nurse. Hollywood, that formative region of all that's good in America has already accepted the female wrestler, and the male exotic dancer. Can high school counselors be far behind?

3. Remediation

This word has become accepted to mean bridging the less qualified up to acceptable educational levels. However in this connection it has a different meaning; more like restitution. We must provide instruction to return to our female students some of the math learning which was literally stolen from them during their school years. We should supply extra instruction to restore women to that state of mathematical equality that they had as girls. Such a program would remove the affect of this biological bias.

4. Instructional Organization

Since large competitive classes unfairly cater to male likes and female dislikes, attention should be devoted to investigating other possibilities. With all options that American education offers, it should be possible to set up smaller, less competitive learning situations for girls. In fact, for a variety of reasons it may be beneficial to have boys also learn in such environments. As with other fields of endeavor, it would probably be found that so too, learning occurs best in a mixed situation. Only this time, let's include situations more favorable, or at least, less unfavorable to the girls. Let's educate both sexes in both environments. This would probably remove the causes of this biological bias.

5. Teacher Training

Let's make all our teacher candidates aware that there are these subtle educational differences between men and women. Although awareness is necessary to solve the problem, it is not sufficient. Teachers should be taught to operate in both environments. Better still, they should be taught how to blend both into that American goal - the achievement of a middle ground. Not only would such training remove both cause and effect of this genetic bias, it would also smooth the transition, and minimize the readjustments of all concerned.

It is hoped that this paper by presenting the problem in such detail has taken a step in the direction of the solution. To paraphrase the astronaut, "One small step for people, but a giant step for people-kind."

RECOMMENDATIONS

Three final points may be appropriate here:

1. Further "hard research" concerning causes for the superiority of male achievement on SAT math scores is necessary.
2. Attempts should be made to create less competitive, more cooperative environments for testing, if not learning.
3. In view of the uncertainty of the cause of the male superiority on math achievement scores, extreme caution should be exercised in the use of such scores as a basis for overall evaluation, and especially for comparision.

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