A REPORT OF INTERVIEWS AND OBSERVATIONS AND THE RESULTING PROJECTS SUPPLEMENT TO ELEMENTARY ALGEBRA SUPPLEMENT TO INTERMEDIATE ALGEBRA

Sabbatical Leave Report
Fall Semester, 1988-1989

Submitted by

Ernest D. Mohnike
Instructor of Mathematics
Mt. San Antonio College

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Purpose of Leave

After 27 years of teaching Mathematics at Mt. San Antonio College I felt that some serious thought needed to be given to reaching students that were not now succeeding in my classes. Too many times students that were working hard and appeared to have good ability were dropping or receiving a poor grade. I undertook this leave to try to discover how other teachers were coping with this problem. Here I was looking for new teaching approaches, including motivation, curriculum, and specific teaching methods.

Secondly, I undertook a project to produce supplements for elementary and intermediate algebra that would be useful to me, my colleagues and our students. Using the ideas collected in talking to a variety of teachers and administrators, and from books reviewed, I have prepared these supplements. These are designed to bridge the gaps that exist between the lecture, the book and the student. They include general approaches to the study of algebra as well as special techniques and examples. There are sample tests and topics not covered, or inadequately covered, in the text. They are designed to be used by other teachers but their principal use is to improve my own teaching. These supplements will be made available on computer disks, so that they may be changed to suit the person using them.

Report on Interviews and Visits to Schools

In the fall of 1988 I visited schools in southern Australia and New Zealand where I interviewed teachers and other professionals. I visited secondary schools, universities, teacher colleges, libraries, and Departments of Education. The following is a report of those visits.

School Visited: Burwood Girls High School
Queens St.
Croydon, New South Wales
Australia

Department Chairperson: Mia Kumar

Notes on visitation: I interviewed Mia Kumar on the workings of the High School and on education in Australia. We talked about my project and she suggested the use of flow charts as a supplemental tool that might be useful. I then visited four classes where I observed and interviewed the teachers. The calculus class I visited had no teacher, providing me with the opportunity to teach the class and talk to the students about a variety of topics. They were very interested in how the American university system worked and if it might be possible for them to study here. In all I attended classes studying elementary algebra, calculus, geometry and general math. I also attended a department meeting where I was able to talk to the mathematics staff about my project.

Useful Ideas: One class that was studying geometry met on the playing field and was measuring and estimating lengths and areas using the metric system. The class estimated the distances and then measured to check their estimate. They were then asked to use the Theorem of Pathagoras to check results.

Comments: Some classes were taught without the use of texts. The curriculum was not divided into algebra, geometry, statistics, and calculus but was integrated at each grade level. This meant that students began the study of statistics, for example, in what would have been the eighth grade and then did additional units each year while in high school. I found this system common to all schools I visited in Australia and New Zealand.

School Visited: Petersham Girls High School
Gordon St.
Petersham, New South Wales
Australia

Department Chairperson: Keven Ford

Notes on Visitation: The day that I visited Petersham High School there was a student strike in progress. This caused most of the classes to be low in attendance and caused many of the teachers to alter their plans. This school is located in a low economic area near Sydney. This caused some difficulty in recruiting mathematics teachers. The first class I visited was taught by a teacher trained to teach English but the others were fully qualified mathematics teachers. I visited three classes and a department reception was held to introduce me to the mathematics faculty. No useful information for the project was obtained but I got some useful teaching ideas.

Useful Ideas: In the class taught by the English teacher I was asked to answer questions for the class. When I wrote an expression on the board the class laughed as I had written $\times$ instead of $\mathfrak{x}$, for the variable name. The letter $\times$ is used for the times operator and the letter $\mathfrak{x}$ for the variable name. I have noted this usage in the work of many of my students of foreign origin.

Comments: The students were allowed to choose their own subjects or tracks but 80% chose to take mathematics through the eleventh grade. This meant most students there were exposed to some calculus, putting them far ahead of where average American high school students finish mathematics. Their top students finished in high school what would be the first year of our college calculus. The rigor in their program appears to be about at the level of our college course.

School Visited: Sydney College of Technical Education
Sydney, New South Wales
Australia

Department Chairperson: Leslie Short

Notes on Visitation: This school is post-secondary education for the bottom third of the students coming from the secondary school. Although it is possible to transfer to the university from the technical school it is quite unusual. The buildings were old and dilapidated but the education going on seemed first rate. I visited with Mr. Short about my project, the school, and Australian education. I visited classes, the math tutoring area and other departments of the college. The mathematics class that I attended was studying calculus from a practical approach. The instruction seemed good and the students interested.

Useful Ideas: The math tutoring area was staffed only by qualified teachers. I asked about using good students to help tutor but they felt the students would be confused if not given quality tutoring. In one area I visited the program consisted of short term courses making the students ready for a certain job market. For example, they have a six week course in waiting tables. Here, among other things, they taught students to make change.

Comments: The students ranged in age from 18-40 but the majority were at the younger end of the range.

School Visited: Canberra College of Advanced Education
Canberra
Australia

Department Chairperson: Dr. C. Annice,
Director of Elementary Education

Dr. Warren Atkins,
Director of Teacher Education,
Secondary Mathematics

Notes on Visitation: In my interview with Dr. Annice we discussed my project, including uses of flow charts to teach mathematical algorithms. She commented that flow charts are used in the elementary school so that a background is in place for using them with older students. We also discussed error analysis as it might be taught to algebra students. She felt calculators should be used at all levels of education. She said that studies indicate that most of the difficulty students have with algebra stems from deficiencies in reading.

Dr. Atkins felt that most teachers depend too heavily on texts. He did not use one in the secondary mathematics teacher program. We discussed various aspects of my projects. He felt that flow charts should not be more than five steps long.

Useful Ideas: The idea of postponing the concept of long division until after division is taught in algebra seemed promising. When calculators are fully utilized it might be eliminated altogether. Fractions, except for the basic concept of what a fraction is, might also be postponed. Calculating, using fractions, is principally necessary if the metric system is not in use. Fractions are not easily done using calculators. If these topics are cut back or eliminated a great deal more time might be spent on practical problem solving.

Comments: Error analysis (how to find your mistake) would be an interesting subject for some future paper or book. Many things that are being done or proposed for Australian schools depends on the removal of artificial course boundaries. This has been proposed for American schools, but is many years away from being implemented.

School Visited: Christchurch Teachers College
Christchurch
New Zealand

Department Chairperson: Stephen French

Notes on Visitation: We discussed pretesting. Teacher prospects needed remedial work before starting their college teacher-training. This was accomplished with SRA type materials in a math lab environment. He felt math should be fun. Use of projects was recommended in the teaching of mathematics.

Useful Ideas: Flow charts should be less than six steps. The idea of using math projects needs to be explored further.

Comments; I visited the library looking for materials. Books reviewed:

The Math Lab- Theory and Practice
Authors: Keys, Post
Prindle Webber Schmidt

Research on Learning and Teaching Mathematics

Authors: Bell, Kirchemann

School Visited: Burnside High School
Christchurch
New Zealand

Department Chairperson: Bill Alwood

Notes on Visitation: Burnside High School is a large suburban school. It is located in an affluent area and is one of the best in New Zealand. I spent the entire school day visiting different classes and teachers. In the first class I visited the topic was Euler's method of solving differential equations. This topic would not be covered here until a student reached upper division course work in mathematics. The reason for this early coverage was twofold. One, the topic was useful in relatively simple applications and secondly it was easily tied to a computer application. The second class was studying statistics and the material being used was from the Kent Mathematics Project, developed in England. This class was taught using a three person team approach. The second teacher presented a unit on a number system base -2. In a class of students about age 15 the topic was linear programming. The problem being considered was: Given x kilos of seed A and y kilos of seed B to minimize $c = 40x + 80y$ with constraints $x \leq 10$, $y > 80$, $x \geq 0$, and $x + y \geq 150$. The approach was a graphical one and the text material was written by a teacher at the school. Each class I visited in this school, even lower level classes, showed energetic students and teachers using the latest techniques to teach mathematics.

Useful Ideas: The idea that the mathematics courses at MSAC might be integrated came to me at Burnside. There are many problems involved in accomplishing this and it may not be possible until integration is accomplished at the secondary level. A study group is currently looking at this kind of arrangement for American schools and it very well may be on the horizon for us whether we want it or not.

Comments: The success I saw at Burnside left me envious. A great deal needs to be done to increase the competence of students here. The way we have done things before is not going to be good enough for the future.

Department of Education:

James Moule
Mathematics Consultant
Remmington Center
Sydney, New South Wales
Australia

Notes on Visitation: Mr. Moule arranged my visits in New South Wales. We talked at length about the things that were happening in mathematics education in Australia. His approach was curriculum oriented and he believed that with good, well trained teachers and modern materials little else is necessary.

Department of Education:

Andy Begg
Education Office
Mathematics Curriculum Development Division
Wellington, New Zealand

Notes on Visitation:

Mr. Begg provided insight into what was happening in mathematics education in New Zealand. Materials available in the Education Office were provided for me and he gave me additional contacts in various cities. We discussed mathematical symbolism and what might be done to simplify it, thereby making calculators easier to use. He said that calculators are used at all levels in New Zealand. He felt that flow charts could be of some value in teaching mathematics. We discussed error analysis and how it might be used to supplement the teaching of algebra.

SUMMARY AND RECOMMENDATIONS

Mathematics education appears to be on the threshold of far-reaching changes. The integrated program in mathematics that I observed throughout Australia and New Zealand will surely reach into the American classroom and will find its way into the program here at Mt. San Antonio College. The changes will be more far reaching than those caused by the so called "New Math" that swept into the elementary schools a few years ago. The rapidly developing field of mathematics requires a more flexible format in order to implement the ideas required by our society. The current curriculum of the secondary schools has remained remarkably static during the years of the twentieth century. All one needs to do is look at mathematics books fifty or more years old to see how little change has taken place. A trigonometry book from the year 1900 would serve as a text in our current trigonometry with only minor changes. The time has come for us to take a hard look at the topics we teach and develop a curriculum for a new century.

Unfortunately we at Mt. San Antonio College cannot initiate these changes. We are too tied to the secondary schools where we get our students and the universities where they go when they leave us. These changes will need to be the result of developing a new curriculum for the schools in America. Our job is to be ready to implement them when they are ready.

At each interview during my stay in Australia and New Zealand I questioned the teachers and other professionals that I interviewed about the supplements to Elementary and Intermediate Algebra that I was to write when I returned. Although I had laid a groundwork for what I was trying to accomplish through the correspondence I had before the visit, I was disappointed with the information that I was able to obtain. It seems that Mathematics teachers do things much the same everywhere. I did collect a wealth of information that will be useful in teaching my classes but the general question of why otherwise good students so often have difficulty in mathematics is still an open one.

The supplements that you will find in the appendices to this report are my attempt to break through to students that are not now succeeding. Many of the ideas that I thought might be of help and that I spent time developing turned out to add to the level of complexity of the material instead of making it simpler. Flow charts and error analysis are examples of topics that were suggested and discussed at many interviews but were not included in the supplements. To a great extent these are only working documents that I have tried less than a semester. A great deal of change will undoubtedly be required before my original goal of helping the student with good ability and work habits to succeed is achieved.

One area I observed that might serve as a model for a mathematics tutoring area was at Sydney School of Technical Education. Here a full time professional was in charge of the area. The layout, materials, and staff competence made me realize that a great deal needed to be done to make our lab area serve the students as it should. In particular, the computer-assisted instruction that was available for their students was superb.

In my visits to libraries I realized that we need not only books for students but a up to date mathematics library for the faculty as well. In particular, current publications should be in close proximity to promote dialogue on the changes that are before us.

In conclusion I would like to thank the Board of Trustees for providing the opportunity for my sabbatical leave. The travels and visitations that I have made should be of value to me and to my students for years to come.

APPENDIX A

SUPPLEMENT TO ELEMENTARY ALGEBRA

The enclosed disk is to help anyone using these supplements adapt them to their needs. The supplement is written using EXP, The Scientific Word Processor written by Simon L. Smith and Walter L. Smith and published by Wadsworth & Brooks Cole Advanced Books and Software. This program is available on the IBM personal computer in the Mathematics department.

APPLICATION FOR SABBATICAL LEAVE

FALL SEMESTER 1988

ERNEST D. MOHNIKE

MATHEMATICS DEPARTMENT

MT. SAN ANTONIO COLLEGE
Salary and Leaves Committee

APPLICATION FOR SABBATICAL LEAVE

Name of Applicant Ernest D. Mohnike

Address 5069 Roosevelt St. Chino, 91710

Employed at Mt. San Antonio College beginning September 1962

Dates of last sabbatical leave:

From September 1978 To January 1979

Department Mathematics Division Natural Science

Length of sabbatical leave requested:

One semester x
Fall x Spring ____

Two Semesters ____

Purpose of sabbatical leave:

Study ____ Project ____

Travel ____ Combination (specify) Travel-Project

NOTE: Sabbatical periods are limited to contractual dates of the academic year.

Effective dates for proposed sabbatical leave:

From September 1988 To January 1889

and (if taken over a two school year period)

From ____ To ____

Attach a comprehensive, written statement of the proposed sabbatical activity(ies) including a description of the nature of the activity(ies), a timeline of the activity(ies), an itinerary, if applicable, the proposed research design and method(s) of investigation, if applicable.

Attach a statement of the anticipated value and benefit of the proposed sabbatical activity(ies) to the applicant, his/her department or service area, and the College.

Any change or modification of the proposed sabbatical activity(ies) as evaluated and approved by the Salary and Leaves Committee must be submitted to the Committee for reconsideration.

Ernest D. Mohnike
Signature of Applicant

11-30-87
Date

Applicant's Name ______________________________

THE ACKNOWLEDGMENT SIGNATURES REFLECT AWARENESS OF THE SABBATICAL PLAN FOR THE PURPOSE OF PERSONNEL REPLACEMENT. COMMENTS REQUESTED ALLOW FOR RECOMMENDATIONS PERTAINING TO THE VALUE OF THE SABBATICAL LEAVE PLAN TO THE COLLEGE.

APPLICANTS MUST OBTAIN THE SIGNATURES OF ACKNOWLEDGMENT PRIOR TO SUBMITTING APPLICATION TO THE SALARY AND LEAVES COMMITTEE.

ACKNOWLEDGMENT BY THE DEPARTMENT/DIVISION

Signature of Department Chairperson Barbara Crane Date 11/30/87

Comments:

Signature of Division Dean Barbara Crane Date 11/30/87

Comments:

ACKNOWLEDGMENT BY THE OFFICE OF INSTRUCTION

Signature of Asst. Superintendent/Vice President, Instructional & Student Services [signature] Date 11/30/87

Comments:

NOTE: DIVISION DEANS ARE REQUESTED TO SUBMIT A STATEMENT OF RECOMMENDATION REGARDING THE VALUE OF THE SABBATICAL PLAN TO THE COLLEGE, DIVISION/DEPARTMENT, AND INDIVIDUAL, IN CONSULTATION WITH THE APPROPRIATE DEPARTMENT CHAIRPERSON.

****

FINAL ACTION BY THE SALARY AND LEAVES COMMITTEE:

______ Recommend approval to the Board of Trustees

______ Not recommend approval to the Board of Trustees

Signature - Chairperson, Salary and Leaves Comm. Date

Signature - Authorized Agent of the Board Date

myw
10/28/87

PROJECT FOR SABBATICAL LEAVE

BACKGROUND OF THE PROJECT:

During my years of teaching, students in all of my classes have repeated the same sad story. These are students that for the most part are successful in their other classes and work very hard at being successful in mathematics. The story is almost always expressed in the same words and takes two general forms. First I hear, "Mr. Mohnike I can understand it when you do it on the board but I cannot do it when I get home", and second I hear, "Mr. Mohnike I can do the work on the assignments but I do terrible on the tests".

I believe that we have failed to address the real problems of these students. Both the text and the lectures miss essential things that make a student work the problems on the homework and tests correctly. I have developed my own set of ideas on how both the homework and class work might be approached in order that these students might improve their success in mathematics. I have incorporated many of these ideas into my own teaching techniques. What I am lacking is the time to formalize these and other ideas I can find into formal concrete steps that the student can use to overcome his or her difficulties.

THE PROJECT:

I propose two things. First I will research what others might have to say on these general topics. This will include visitations to schools that teach what is generally covered in our elementary and intermediate algebra courses. At these schools I will seek out teachers that have special interests and ideas on these topics. From these teachers and from reading in the general areas of heuristics and teacher education in mathematics I hope to enrich my own ideas on how to improve student success.

Second I will take the collected ideas and write a supplement to the courses of elementary and intermediate algebra. These materials will be a maximum of twenty five pages in length each. These would be much longer if all of the things I found useful were included but to make it short enough so that the students will read it I will limit its length.

AREAS TO BE CONSIDERED FOR THE SUPPLEMENT:

These will be designed to be used primarily near the start of the course.

1. How to approach the lecture.
2. How homework should be approached so as to maximize understanding and retention.

These will be designed to be used as the course progresses.

1. How to prepare for each unit test.
2. General problem solving techniques with examples of how they apply to various algebra topics.
3. Special topics that are not generally included or are inadequately treated in the algebra texts currently being used.

HOW THIS CAN BE UTILIZED AT MT. SAN ANTONIO COLLEGE:

This supplement could be used in several ways:

1. Other faculty members who use the same text could use all or parts of it with their students.

2. Those faculty using different texts might discover some useful ideas for their own classes.

3. It could be used with mathematics tutors to improve coordination with the classroom teachers.

TRAVEL:

Beginning in September, 1988 and extending through December, 1988, I plan to visit teacher-training colleges and universities as well as secondary schools in southern Australia and New Zealand. A list of the particular schools will be presented as soon as finalized. A minimum of six schools will be visited.

Following is an approximate timetable:

AUSTRALIA:

September:	South Australia
October:	Victoria
	Australian Capital Territory
	New South Wales

NEW ZEALAND

November:	South Island
December:	North Island

Return January 1989 to compile results and write classroom supplements.

RESPONSE TO RECOMMENDATION FOR REVISIONS

1. Although the background research for the project could be done here I believe a new perspective needs to brought into our teaching of mathematics. One of the reasons for the choice of algebra as the topic of the project is that it is a topic that is taught in almost all systems everywhere. Australia and New Zealand were chosen because of a desire to travel in this region on my part and because algebra is taught in a variety of types of schools, and to a wide range of ages.

2. The data collected is not intended to be statistical in nature but rather ideas on how to reach the students we are not now reaching. I am hoping to talk to some of the best teachers in the region and perhaps observe their classes. I will also look at texts being used. Notes on what I discover will be kept for the draft of the supplement and sabbatical report. Ideas that appear to be of merit will be included in the supplement. If the ideas work out with my classes they will be kept in the supplement; otherwise other ideas will replace them.

3. Since the type of interview I will be conducting will be informal I will not use any standard type questions. I will ask questions designed to get the teachers I am interviewing to talk about the things that they do with students that are not doing well but are of high ability and motivation. This might include questions like: "What special things do you do to help a student who says that they can do the work at home but do not do well on tests?" or "Are there special

exercises that you assign this type of student?" or "How do you handle some special topic that tends to confuse these students?" Hopefully I will discover those ideas and materials that make his or her teaching special.

4. In talking to members of my department I have found some interest in the ideas of the study. In order to increase the value to Mt San Antonio College I will provide computer disks to those mathematics department members that are interested. This should provide a tool so the material can be easily adapted to their needs.

5. I believe that the material needs to be specialized. The algebra texts suffer from generalization so that many teachers will use them. This supplement will hopefully bridge the gap between the text, the student and myself. For each teacher and each text and for that matter each student the gaps are different.

and ideas on these topics. From these teachers and from reading in the general areas of heuristics and teacher education in mathematics I hope to enrich my own ideas on how to improve student success.

Second I will take the collected ideas and write a supplement to the courses of elementary and intermediate algebra. These materials will be a maximum of twenty five pages in length each. These would be much longer if all of the things I found useful were included but to make it short enough so that the students will read it I will limit its length.

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HOW THIS CAN BE UTILIZED AT MT. SAN ANTONIO COLLEGE:

This supplement could be used in several ways:

1. Other faculty members who use the same text could use all or parts of it with their students.

SUPPLEMENT TO ELEMENTARY ALGEBRA

E. D. MOHNIKE -1990

This supplement is designed to provide a bridge between the textbook, the lecture, the teacher and you. Some of the material will be further explanation of materials in the text and some will be new materials omitted in the text. This material is not meant to be used alone but in conjunction with the text and lecture. You will find that some of the ideas will be general in nature, providing a background in how to study and prepare mathematics problems, and some will be on specific topics from the text.

CONTENTS

SECTION 1: PREPARING DAILY WORK

The work that you prepare on a daily basis, both in class and at home, is designed to teach the fundamental processes of algebra and to provide sufficient practice so that the material will be retained. How many problems you need to work to accomplish this objective will vary greatly from person to person and, unfortunately, just what this number is for you is not known by either me or you. The number of problems that you need to work is something that you will need to decide, at least partially, for yourself. The assignments are designed to provide enough practice for an above average student with adequate background for the course. You may need to work more problems than are assigned in order to achieve your objectives. The way that you prepare your work can greatly effect the amount of information you retain and the time you must use to succeed in the course.

There are a few basic rules that will help in the preparation of assignments.

Rule 1. Copy the problem correctly.

Rule 2. Decide what type of problem you are solving.

Rule 3. Plan what you need to do for the problem.

Rule 4. Work down, not across, the page.

Rule 5. Check each step before going on to the next.

Rule 6. Avoid distractions midway into a problem.

Rule 7. Don't bog down. Leave problems that cause difficulty and come back to them.

Rule 8. Check your answer. If it is wrong put an X on the problem indicating you missed it on the first try.

Rule 9. Look for your error. Circle or otherwise indicate what caused the error. Correct the error or work the problem over.

These rules are general guidelines and are designed to develop skills that will help you prepare for your tests. Some of these rules will be covered later in the supplement but a few comments are necessary at this time.

Rule 1-3. Covered in detail later in the supplement.

Rule 4. It may appear that paper can be saved if you work across but this will lead to errors in your work and it makes the problems difficult to check for both you and your teacher. **WORK DOWN!**

Rule 5. It is far easier to find problems when they occur than to try to find them later. A small error in a step will cause the rest of the problem to be wrong. When this happens a simple problem may be made much more difficult and require a great deal of time. At each step check your work. If it appears correct put a check beside it. It might appear that it would take more time to do this but in reality less time will be required. Checking is more effective if you do not try to do exactly the same step over. For example, the step was to factor -3 out of $-3x+6y$ yielding $-3(x-2y)$. It would be better to distribute the -3 to see if it resulted in the original expression than to try to factor the first line again. Sometimes because of difficulty in reversing the steps you may need to check by doing the step over.

Check each step! If correct put a check beside it.

Rule 6. Many mistakes are caused by a loss of concentration midway through a problem. Give yourself enough time and don't look up until the problem is finished.

Rule 7. Covered in detail later in the supplement.

Rule 8. Remember the daily work is designed to help you succeed on the tests. You need to work the problems daily in such a way that when the test comes the problems can be worked in the same manner as practiced on the homework. The difficulty is that usually the answers are available for the daily work but on tests you do not have them. In order to compensate for this you must do the daily work with maximum effort directed at getting the problem right the first time. In order to accomplish this I want you to put an X by any problem you get wrong on the first try, even though you subsequently correct it. Remember using rule #5 you are checking each step as you go so many problems will be correct on the first try. If you get many problems wrong at first, in any set, you need to continue working problems until you get them right the first time.

Get them right the first time!

Rule 9. Errors tend to reoccur. If you use the homework for preparing for tests and you have marked the type of errors that you have made, you can avoid making them again when they count the most.

SECTION 2: PROBLEM SOLVING

We will now look at some general ideas on solving problems in algebra. Probably the most important reason for taking algebra is the problem solving skills that you should develop. When you encounter a problem in algebra there are certain steps you should take whether the problem occurs in daily work or on a test.

1. Categorize the problem.
2. Develop a plan.
3. Work the problem according to the plan.
4. Check your work.

The first step, categorize the problem, is the step that is the most difficult to get from the book. This is because the method that is most often used by students to solve problems is to look back in the book to see how it is done. This, coupled with the fact that the book groups problems that are alike in the same section, leaves the student (you) with little practice in categorizing the problem. You can overcome this problem by carefully studying not only how to solve the problems but also paying close attention to the type of problem that is likely to occur. For example, the topic of fractions might include problems of the following types: reducing, multiplying, dividing, adding, solving equations, complex fractions and word problems. You may know how to solve equations and add fractions but if you use the plan for one to do the other the result will be poor indeed. Before you start any problem you must think carefully about what kind of a problem you are working. The instructions for the problem give the key to categorizing it correctly. It will never say solve if what is required is to add. *You* must look at these key words and associate the correct method with them. Some of these words will not have a clear meaning at first. "Simplify" is a common word that often confuses students. How simple is simple? When are you finished? The answer to these questions will be covered in the text but one thing is certain, "solve" and "simplify" mean something different. Consider the following example:

Simplify $4-4(3-2x)$ "Simplify", in this case, means remove the parentheses

$4-12+8x$ ✓ and combine like terms.

$-8+8x$ ✓

$8x=8$? What happened here?

$x=1$ The instruction "simplify" was confused with the instruction "solve". You find the value for x only if the instruction is solve. Do not add an = to a problem that does not have it.

Categorize the problem. Develop a plan. Follow the plan!

Correct solution $4-4(3-2x)$

$4-12+8x$ ✓

$-8+8x$ ✓ Now the problem is finished. Note you cannot combine the -8 and the $+8x$. The terms are not alike. No exact method of checking is available but if time permits values could be chosen for x and substitutions made into the problem and answer to see if the value is changed. If it has then you have made an error in your work.

In spite of careful preparation, there will be times both on daily work and on tests that you will not be able to solve a problem. You draw a blank. What do you do? Of course you can get help from the Math Lab, a teacher or a friend and at times that is exactly what you should do, but that won't help on the tests. Remember, the skills you develop in your daily work will help you on the tests. You need to practice problem solving in your daily work. If you depend on outside help and problems being worked in class you will miss this necessary step in problem solving. **Work the problems yourself.**

There are some basic questions that you should use to improve your problem solving skills.

A. Understanding the problem.
1. What is the unknown?
2. What are the data?
3. What is the condition?
4. Is it possible to satisfy the condition?
5. Did you draw a figure?

B. Devising a plan.
1. Have you worked a similar problem before?
2. What connection is there between the data and the unknown?
3. Can you categorize the problem into one of the basic types?
4. Have you used all of the data?

C. Carrying out the plan.
1. Did you carry out the plan completely?
2. Did you check each step?

In algebra you will almost never be asked to solve some problem without seeing one of the same type before. You must identify the type and know how to solve it. The key words used in the problem will help you plan your solution. The best advice I can give is to do something. Simplify, add and reduce fractions, combine like terms and remove parentheses. In **equations** you can add, subtract, multiply, square (check for extraneous roots), take the square roots ($\pm$) of both sides. In word problems choose a letter or letters for the unknowns. **Try something!**

The objective of the course is for you to learn algebra. The principal way that we measure this is through the tests in the course. This includes quizzes, unit tests and the final exam. Students have often said "I can do the problems on the homework but I do not do well on the tests". It is the objective of this section to help with this problem. This will be done in two parts:

1. Preparing for tests.
2. Taking tests.

PREPARING FOR TESTS

If you have prepared your daily work carefully you should have the problem of preparing for any test ninety percent accomplished. This, of course, does not mean you would get 90 % on the test. With a 10% error factor you might miss all of the problems. In fact, I have observed that a few small hints given at the right time could improve student scores dramatically. What you need to do is learn to provide your own hints.

The first thing you need to do is to review all types of problems that the test will cover. This should especially include problems that you worked without difficulty in the homework. These are your bread-and-butter problems and if you miss these "easy ones" your grade will be terrible.

Study the easy ones!

Work problems of each type until you get them right the first time. Check each step. Review your homework. Pay particular attention to the places that caused you to miss problems the first time. Look for the circled parts. This is where the time you took to indicate what caused you trouble in previous problems will pay off. Remember, on the test the answers will not be available. The habits you develop in your study will effect how well you do on the test.

Practice categorizing your problems. Use the practice tests in the book to do this. Before you start a problem think: "What type of problem is it and what plan will work to find the solution?". If you must go back into the text to do problems in the chapter tests you need more study.

Now you are ready to take the practice test in this supplement. Before you start let me remind you that these tests are designed to find out if you are ready for the test and you should take it with this in mind. Give yourself an entire hour. Put your book away. Work each problem carefully. Remember to check each step. Leave any problem you cannot work until later. After you finish check the answers. Play it straight. Only you will know how you did. Ask yourself the question, "Did I do well enough so that I am ready for the test?". If the answer is no then you need more preparation. Remember, the Math Lab and I will willingly help you with any type problem that you cannot work, but you must give yourself enough time to get it.

TAKING THE TEST

You are now prepared for the test. This does not assure a great grade. You must take the test in such a way that what you know is communicated to me. There are a few basic ideas that will help.

1. Work each problem just as you practiced. Categorize, plan and check.
2. Work the easy problems first. Don't hurry! Remember these are important to your grade. Leave room for any problem you skip.
3. If a problem is taking too long, leave it! Come back if time permits.
4. Show your work. If you are in doubt about whether a step should be included, put it in. Even if you miss the answer there may be some partial credit and these points count. Don't erase a problem and leave me nothing to grade.
5. Neatness counts. You need it to work the problems and I need it to grade accurately.

SECTION 4: CHOOSING AND USING YOUR CALCULATOR

A calculator will help you very little and could hurt you a lot in elementary algebra. You must learn to use it effectively and know when to use it, then it can be of some use in a limited number of problems. If you use it on tests and have not practiced with it, your grade will suffer in three important ways. First, you will take time from the real business of doing algebra. Second, you will miss answers because of doing the calculations wrong. Third, you will leave out steps that may be necessary in the grading. Still, because it can help students who use it correctly and knowing how to use it is a skill that can be valuable in many situations, I allow and encourage it's use.

There are two basic types of calculators that can be of use to you in algebra. The first is the type that uses what I will call adding machine logic and the second uses the logic we most often use in algebra. Consider the problem $2+3\cdot5=$?. If you enter this problem just as it appears into a simple calculator using adding-machine logic you will get 25 for the answer but if you enter it into one that has algebraic logic you should get 17 as the answer. Try this with any calculators you may have or are considering buying. The reason for getting 17 with some calculators is they perform the order of operations in the manner that we do in algebra. That is, in this problem they multiply before they add. Although either type will work for you, in this course the type with the algebraic logic will be simpler to use. The other consideration might be price. Most calculators, especially those you carry with you on a daily basis, have a limited useful life, therefore: do not spend too much money.

Look now at a few types of calculator problems that might be useful in algebra. First let me say that the notation that we use in algebra does not always work well with your calculator. This is because many things in algebra are implied by the notation, whereas with your calculator everything must be entered.

For example, $3(4+5)=$ means multiply the sum of 4 and 5 by 3. The addition is done first as it would be on your calculator but the times operation is implied by the notation. To work this problem on your calculator you need to put in the multiplication operator. That is, you need to think of the problem as $3\cdot(4+5)=$. Put in the times operator.

The problem $\frac{3+4}{5+7}=$ uses the horizontal line to indicate both division and grouping. That is, you must think $(3+4)\div(5+7)=$ in order to use your calculator directly. This problem also brings out another problem with the use of calculators. The calculator gives an answer like .583333 and what we usually want is $\frac{7}{12}$. You must learn when decimal answers (approximate) or fractional answers (exact) are expected.

If you want to evaluate x^2-3x+4 when $x=-2$, with your calculator, you may find it easier to use the memory function provided. In order to evaluate (key word), using an algebraic logic calculator, first enter 2 and then push the $\pm$ key to get -2. Push the M key to put -2 in memory. Then calculate the expression using the following steps:

Square, $-$, 3, $\times$, recall M, $+$, 4, $=$. You did not need to recall first because the -2 was already in your calculator register.

There are many types of calculators. In order to use yours consult your manual or see me. <u>If you do not practice with your calculator leave it at home on test day</u>.

CALCULATOR PRACTICE PROBLEMS

1. $3 \cdot 7.25 + 8 \cdot 1.25 =$

2. $\dfrac{5.25 - 3.75^2}{4.3^2 + 2(1 - 3.75)} =$

3. $X = -2.4$ Find $x^2 - 3.4x - 6.42$

4. $y = 3x + 5$ Find y when $x = -3.7$

5. -34.27^2

6. $(-1.234)^2$

7. $\sqrt{2.5^2 - 1.25^2}$

8. $\left(\sqrt{3.4} + \sqrt{5.6}\right)^2$

SECTION 5: AN ALTERNATE METHOD OF SOLVING WORD PROBLEMS

Many students have trouble solving word problems. This type problem is especially important if you have to apply mathematics in the solution of a problem. Any real problem that you can describe in words becomes a word problem. If you cannot solve them you cannot apply algebra. If you cannot apply algebra it will do you little good.

The method we will examine here will be illustrated by the following example.

> If four times the smallest of three consecutive integers is added to the largest the result is 112. Find the integers.

Solution: We will start this method by guessing the answer. Then we will check to see if the guess is correct. We will try to guess as little as possible and remember that we are not trying to guess the answer but to write an equation that will lead to the solution. Let's try 25 for the first number. Is this correct? Try it. ··· I get 127. Four times the first plus the third: $4(25) + 27 = 127$. Most of us can tell whether the answer is correct even if we cannot work the problem. How can this reasoning be used to solve the problem?

Let's guess again: 23. Wait, before you check to see if it is correct write the other numbers in terms of the first. That is, instead of 23, 24, and 25 write 23, 23+1, and 23+2. Wait to do the arithmetic. Now check using this form.

$4(23)+(23+2) \stackrel{?}{=} 112$ #1

Now the arithmetic.

$4(23)+25=$ ✓

$92+25 =$ ✓

$117 \neq 112$. Wrong answer but then we were not trying to guess the answer. Now guess again, but this time guess x for the first number. Then the three numbers are x, x+1, x+2.

The check: $4x+(x+2) = 112$ #2

$4x+x+2=112$ ✓

$5x+2 = 112$ ✓

$5x+2-2 =112-2$ ✓

$5x=110$ ✓

$x=22$, $x+1=23$, $x+2 =24$ and $4(22)+24=112$ ✓

Look at the lines numbered #1 and #2. We used x in exactly the same way we used the numbers when they were guesses. For more information on this method check with the Math Lab for a video tape on this topic: An Alternate Method of Solving Word Problems.

A FURTHER NOTE ON WORD PROBLEMS

In algebra there are only a few main types of word problems. Regardless of the method you choose to use in solving them, there is one more important fact to remember. You need to work on one type at a time until you know how to do that type. For instance, there may be seven rate problems in a particular chapter in the book. Only three may be assigned but you may need to work them all if that is what it takes to develop your skills in that type problem. When word problems appear in a later chapter you will be able to work rate problems and then you will be able to concentrate on some other type of problem. **Remember, focus on one type of problem until you have it mastered.**

SECTION 6: WHERE YOU CAN GET HELP

Where do you go when you need help and when do you need it? Generally on a day to day basis most of your help will come from the text. Every type of problem that you will be expected to work is covered there, both in theory and by example. Answers to the odd numbered problems are in the appendix. You need to work with the text until you are familiar with it. Your first source of material will be your text. Assignments will be made from problems chosen from it. After you have attended the lecture and read the book you should be ready to start the homework. Don't skip a step. Many students try to work the homework without adequate background by following the examples in the text, but this will not be enough information for you to work the problems effectively.

Start any assignment slowly. Remember, not enough problems may be assigned for you to catch on to an idea. *If you are having difficulty with a problem go back and work some more easy ones.* Look for the step that is causing the trouble. If you cannot find the trouble then go on. Don't bog down. You need to try all of the problems assigned before the next class. If you have a significant number of problems you are not able to solve, you need help. Don't leave them for the teacher. You need to solve them yourself. Seeing them on the board will reinforce them but if you have not worked them before they will be reduced to just another example. <u>You need to spend two hours outside of class for each hour in class.</u> If the homework is taking significantly more time than this something is wrong.

You need help. Where do you get it? Three main resources are available here at the college. First see your teacher, that's me. If you need help come and see me first. I keep regular office hours every week and if these do not fit your schedule I will arrange others. I want to help you. Give me the first chance. Come to me early and often. I will be able to look at your work and decide what is going wrong better than anyone else. Working problems in my office can get you going. Don't wait for a bad test result. My office is 239E. The regular office hours are posted on my door and announced in class.

If you need more than this we also have the Math Lab in 7-126. This facility is open every day from 8 a.m. to 10 p.m. except Friday when it is open from 8 a.m. to 5 p.m.. Some students find that working the majority of their homework there is about the only way to accomplish it in the two hours allotted.

In the math lab the most important resource is the tutors. Please remember that they are not trained teachers. Don't expect them to do either of our jobs. They are not supposed to work the problems for you. You ask: "Where am I going wrong?". Not: "Work this problem for me". If new methods are presented to you in the lab ask the tutor to do it by the methods of the book. I use this method, you use it. They can help you with it. If not, see another tutor or see the person in charge of the lab. She is trained and can give you first class help.

The best hours for the lab are in the afternoon or evening when there is not a crush of students. If you need more than 5 or 10 minutes help an hour you are not using the lab correctly. You need to work the problems yourself. Also available in the lab are video tapes on the topics of algebra. These may be useful to supplement the lecture.

Tutors are also available in the study skill area of the library. As in the lab these are good students that can help you if you give them a chance. Shop around early. Find a tutor you can work with before some crisis exists. Don't wait until the test. By then it may be too late.

SECTION 7: WHAT TO DO IF YOU THINK YOU SHOULD DROP

Sometime in any course it may appear to you that you should drop. There are certain things you should do before you do this. First see your instructor. You may not be as bad off as you appear or there may be something that we can do to help the situation. **Come see me!**

Students often make the decision to drop unconsciously. It often starts with the decision to miss class because you are not prepared. This is often the time when coming to class is the most necessary. You will just get further behind. Get help! Go to the Math Lab. Since you have invested this much time, do not drop without careful thought.

If you decide to drop, take note. In the first five weeks you may be able to drop back to an easier course. This will not help if you are not willing to do the work but if you are trying to do well and can't, see me.

Drop back, don't drop!

State law prohibits dropping after the thirteenth week. If you stop attending after that week you will fail the course.

No drops after the thirteenth week! Take the final. Things may be better than they seem.

SECTION 8: COMMENTS ON TEST 1

The first test is important as it can set a trend for the entire course. You may have heard the expression "math anxiety". This describes a Math 51 student who does poorly on the first test. You may have taken algebra sometime in the past and be in a position where you "know this stuff" and do not need to study. You must be especially careful as some small things can trip you up at this early stage. Problems like $2+3\cdot5$ can cause difficulty if you do not consider the order of operations and do the addition first. The problem $3+2(4+x)$ is the same. You must remove the parentheses first. You cannot add the 3 and the 2 first.

You must also know the difference between -2^2 and $(-2)^2$. In more complicated problems simplify by first removing the innermost parentheses .

Example: Simplify $3+5[\{2-4x\}-2x]$ Think what does simplify mean?

$3+5[2-4x-2x]$ ✓

$3+5(2-6x)$ ✓

$3+10-30x$ ✓

$13-30x$ ✓ Notice since we were simplifying we did not find x.

Example: Solve $2-4[1-3x]=-5$

$2-4+12x=-5$ ✓ Get the signs right!

$-2+12x=-5$ ✓

$-2+12x+2=-5+2$ ✓ Add + 2 to both sides.

$12x=-3$ ✓

$x=-\frac{3}{12}$ ✓ Divide each side by 12.

$x=-\frac{1}{4}$ Since we were solving we found the value of x.

Check: $2-4[1-3(-\frac{1}{4})]$

$2-4[1+\frac{3}{4}]$

$2-4[\frac{7}{4}]$

$2-7$

-5 ✓

COMMENTS on test 2:

It is very important not to mix up the following types of problems.

Example: Add and simplify

$$\frac{1-x}{x^2-x}+\frac{2x}{1+x^1}-\frac{2-x}{x}$$

$$\frac{1-x}{x(x-1)}+\frac{2x}{1+x}-\frac{2-x}{x} \quad \checkmark \quad \text{Factor and reduce each fraction.}$$

$$\frac{-1}{x}+\frac{2x}{1+x}-\frac{2-x}{x} \quad \checkmark \quad \text{Find LCD. } (1+x)\cdot x$$

$$\frac{-1(1+x)}{x\cdot(1+x)}+\frac{2x\cdot x}{(1+x)\cdot x}-\frac{(2-x)(1+x)}{x\ (1+x)} \quad \checkmark \quad \text{Put in the missing factors.}$$

$$\frac{-1-\ x^2\ +2x^2-2-x+x^2}{x(1+x)} \quad \checkmark \quad \text{Watch the signs!}$$

$$\frac{2x^2-x-3}{x(1+x)} \quad \checkmark\text{Keep the denominators. It's not an equation.}$$

Example: Solve

$$\frac{1}{x-2}+\frac{2}{x+1}=\frac{1}{x-2}$$

Again find the LCD but this time multiply each side by it and reduce.

$$(x-2)(x+1)\frac{1}{x-2}+(x-2)(x+1)\frac{2}{x+1}=\frac{1}{x-2}(x-2)(x+1) \quad \checkmark$$

$$+x+1+2x-4\ =\ x+1 \quad \checkmark$$

$$3x-3=x+1 \quad \checkmark$$

$$2x=\ 4 \quad \checkmark$$

$x=\ 2$ Since 2 makes the denominator of the original fraction 0, we have no solution.

Example: Add and simplify- You do this one.

$$\frac{x}{x-2^1}-\frac{2}{2x-x^2}+\frac{1}{x^2}$$

Be sure to use the LCD.

COMMENTS ON TEST 3

Problems involving radicals can cause you difficulty. The general rule is: Isolate one of the radical terms on one side of the equation then square both sides. Keep in mind the difference between squaring a binomial and a monomial.

Example: Solve $\sqrt{x-2} + 2 = \sqrt{x}$

$$\sqrt{x-2} = \sqrt{x} - 2 \quad \checkmark \quad \text{Now square both sides.}$$

$$\left(\sqrt{x-2}\right)^2 = \left(\sqrt{x} - 2\right)^2 \quad \checkmark$$

$$x - 2 = x - 4\sqrt{x} + 4 \quad \checkmark$$

$$-6 = -4\sqrt{x} \quad \checkmark$$

$$3 = 2\sqrt{x} \quad \checkmark \quad \text{Square both sides again.}$$

$$9 = 4x \quad \checkmark$$

$$x = \frac{9}{4} \quad \text{Now it must be checked.}$$

Check: $\sqrt{\frac{9}{4} - 2} + 2 = \sqrt{\frac{9}{4}}$

$$\sqrt{\frac{9}{4} - \frac{8}{4}} + 2 = \frac{3}{2} \quad \checkmark$$

$$\frac{1}{2} + 3 = \frac{3}{2} \quad \checkmark$$

$$\frac{7}{2} \neq \frac{3}{2} \quad \text{Therefore there is no solution.}$$

REMEMBER DO NOT SQUARE BOTH SIDES IN THE CHECK.

COMMENTS ON THE FINAL:

Look over the solution of the following problem. It combines material from several chapters. You should be able to solve it.

Example: Solve

$$\frac{1}{x-2} + \frac{2}{x+1} = \frac{1}{2}$$

Again find the LCD but this time multiply each side by it and reduce.

$$2(x-2)(x+1)\frac{1}{x-2} + 2(x-2)(x+1)\frac{2}{x+1} = \frac{1}{2}2(x-2)(x+1) \quad \checkmark$$

$$+2x + 2 + 4x - 8 = x^2 - x - 2 \quad \checkmark$$

$$x^2 - 7x - 8 = 0 \quad \checkmark$$

$$x = \frac{7 \pm \sqrt{33}}{2} \quad \checkmark$$

Mathematics 51	SAMPLE TEST	Name____________
Examination 1	USE SEPARATE PAPER	Section__________

1. 10	1. If $x = 8$ and $y = 2$, find the value of $y + x$.
2. 9	2. If $x = 2$ and $y = -7$, find the value of $x - y$.
3. 9	3. If $x = 2$ and $y = 3$, evaluate $-y^x$.
4. -9	4. If $x = -4$, $y = -3$, and $z = -2$, evaluate $x + y + z$.
5. Assoc. prop. of +	5. State the property of the real numbers that justifies the statement: $(x + y) + z = x + (y + z)$
6. $z = 3$	6. Solve: $9z = 27$
7. $p = \frac{i}{rt}$	7. Solve for p: $i = prt$
8. 160 calories	8. A piece of cake and a glass of milk together contain 660 calories. The cake has 340 more calories than the milk. How many calories are in the glass of milk?
9. \$50	9. Sally gave \$10 more to charity than Bob did, and Bob gave \$30 more than Maria. If their total gifts were \$220, how much did Maria give?
10. -2 o======>	10. Solve the inequality and graph its solution: $x^2 + 18 > x(x - 9)$
11. 7	11. Simplify: $7a^0$. **Write answer without using parentheses or negative exponents.**
12. 16	12. If $P(x) = 2x^2 - 3x + 2$, find $P(-2)$.
13. $2w^2 + 6w + 1$	13. If $P(x) = 2x^2 + 6x + 1$, find $P(w)$.

14. $3x^2 + 11x + 9$	14. Subtract: $\begin{array}{r} 5x^2 + 9x + 8 \\ \underline{2x^2 - 2x - 1} \end{array}$
15. $6x^2 + 5x - 6$	15. Find the product: $(2x + 3)(3x - 2)$
16. yes	16. Is -1 a solution of $x^2 - 8x = x^2 - 2x + 6$?
17. $x = -5$	17. Solve: $-3x = 15$
18. $y = -12$	18. Solve: $\frac{y}{6} = -2$
19. $x = 1$	19. Solve: $9x + 3 = 12$
20. $3x + 2$	20. Simplify: $5x - 2(x - 1)$
21. $5x^2 + 4x - 25$	21. Simplify: $5^2(x^2 - 1) - 4x(5x - 1)$
22. $x = 0$	22. Solve: $5x = x$
23. $x = -45$	23. Solve: $8(3x - 5) = 5(5x + 1)$
24. $x = 3$	24. Solve: $\frac{4x - 9}{3} = 3x - 8$
25. $r = \frac{i}{pt}$	25. Solve for r: $i = prt$
26. $r = \frac{A - P}{Pt}$	26. Solve for r: $A = P + Prt$
27. 24	27. The sum of two consecutive even integers is 50. What is the smaller integer?
28. 20 ft	28. A 35-ft rope is cut into three pieces so that the second piece is twice as long as the first piece, and the third piece is twice as long as the second piece. How long is the third piece?
29. −2 o======>	29. Solve the inequality and graph its solution: $x^2 + 18 > x(x - 9)$
30. −8●======o−5	30. Solve the inequality and graph its solution: $-5 \leq 5(x + 7) < 10$

Mathematics 51 SAMPLE TEST-SHOW YOUR WORK Name____________________

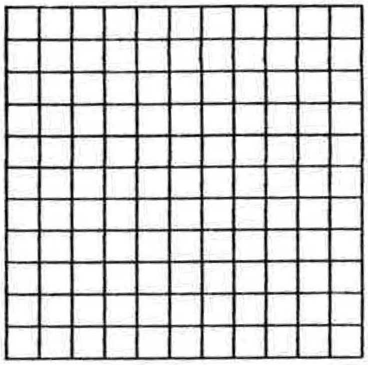
Examination 2 USE SEPARATE PAPER Section________________

Answer	Problem
1. $2^2 \bullet 3 \bullet 5 \bullet 7$	1. Find the prime factorization of 420.
2. $2y^2(x^2z + 2xz - 3)$	2. Factor completely: $2x^2y^2z + 4xy^2z - 6y^2$
3. $(x^2+4)(x+2)(x-2)$	3. Factor completely: $x^4 - 16$
4. $y(3x + 1)(x + 1)$	4. Factor completely: $3x^2y + 4xy + y$
5. $2(a+2)(a^2-2a+4)$	5. Factor completely: $2a^3 + 16$
6. $(x + 3)(a + b)$	6. Factor completely: $3a + 3b + xa + xb$
7. $x = \frac{3}{2}, 5$	7. Solve: $2x^2 - 13x + 15 = 0$
8. 4 feet	8. A rectangle is 2 feet shorter than twice its width, and its area is 24 square feet. What is the width?
9. $\frac{3x - 1}{x - 1}$	9. Simplify: $\frac{3x^2 - 7x + 2}{x^2 - 3x + 2}$
10. $\frac{x - 3}{a + 1}$	10. Simplify: $\frac{ax - 3a - x + 3}{a^2 - 1}$
11. $\frac{y^3}{2}$	11. Perform the indicated operations and simplify: $\frac{5xy^2}{x^2y} \bullet \frac{xy^3}{10y}$
12. y	12. Perform the indicated operations and simplify: $\frac{y^2 - 4y - 5}{y^2 - 2y - 3} \div \frac{y - 5}{y^2 - 3y}$
13. $\frac{-11x + 6}{x(x - 3)}$	13. Perform the indicated operations and simplify: $\frac{3x - 2}{x} - \frac{3x}{x - 3}$
14. $\frac{2y^2 + x^3}{2y^2 - 3x^2}$	14. Perform the indicated operations and simplify: $\dfrac{\frac{2x}{x^2} + \frac{x}{y^2}}{\frac{2x}{x^2} - \frac{3x}{y^2}}$
15. $x = -12, 5$	15. Solve for x: $\frac{10}{x - 3} - \frac{1}{3} = \frac{14}{x - 2}$

Mathematics 51 SAMPLE TEST-SHOW YOUR WORK Name________________

Examination 3 USE SEPARATE PAPER Section________________

1.

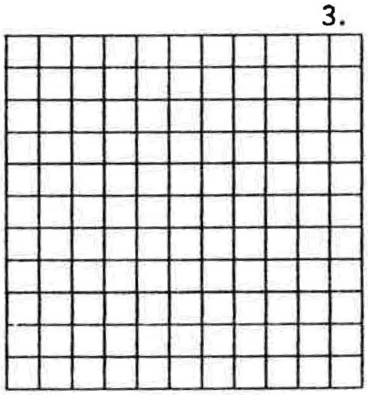


1. Graph the inequality: $y \leq -x + 1$

2. $a - b$
____

2. If $f(x) = x + 2$, find $f(a) - f(b)$.

3.



3. Solve the system by graphing.

$$\begin{cases} x - y = -2 \\ x + 2y = 13 \end{cases}$$

4. $(-2, 1)$
____

4. Solve the system of equations by substitution.

$$\begin{cases} 2x - 3y = -7 \\ 3x + y = -5 \end{cases}$$

5. $(0, 4)$
____

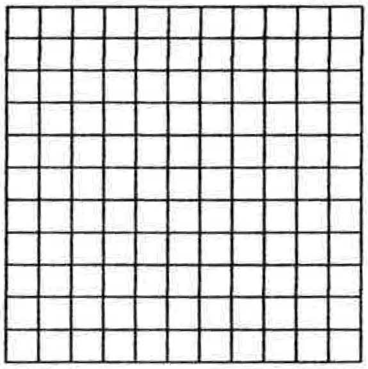
5. Solve the system of equations by addition.

$$\begin{cases} 2x + 3y = 12 \\ 3x - 2y = -8 \end{cases}$$

6. 100 tickets

6. At a movie theater adult tickets cost $5 and student tickets cost $3. One day the total admissions were 450 persons and the receipts were $1550. How many adult tickets were sold?

7.



7. Solve the system of inequalities.

$$\begin{cases} x + y \leq 5 \\ x - y \geq 1 \end{cases}$$

APPENDIX B

SUPPLEMENT TO INTERMEDIATE ALGEBRA

SUPPLEMENT TO INTERMEDIATE ALGEBRA

E. D. MOHNIKE -1990

This supplement is designed to provide a bridge between the textbook, the lecture, the teacher and you. Some of the material will be a further explanation of materials in the text and some will be new materials omitted in the text. This material is not meant to be used alone but in conjunction with the text and lecture. You will find that some of the ideas will be general in nature, providing a background in how to study and prepare mathematics problems, and some will be on specific topics from the text.

CONTENTS

SECTION 1: PREPARING DAILY WORK

The work that you prepare on a daily basis, both in class and at home, is designed to teach the fundamental processes of algebra and to provide sufficient practice so that you will remember the material. How many problems you need to work to accomplish this objective will vary greatly from person to person and, unfortunately, just what this number is for you is not known by either me or you. The number of problems that you need to work is something that you will need to decide, at least partially, for yourself. The assignments are designed to provide enough practice for an above average student with adequate background for the course. You may need to work more problems than are assigned in order to achieve your objectives. The way that you prepare your work can greatly effect the amount of information you retain and the time you must use to succeed in the course.

There are a few basic rules that will help in the preparation of assignments.

Rule 1. Copy the problem correctly.

Rule 2. Decide what type of problem you are solving.

Rule 3. Plan what you need to do for the problem.

Rule 4. Work down, not across, the page.

Rule 5. Check each step before going on to the next.

Rule 6. Avoid distractions midway into a problem.

Rule 7. Don't bog down. Leave problems that cause difficulty and come back to them.

Rule 8. Check your answer. If it is wrong put an X on the problem indicating you missed it on the first try.

Rule 9. Look for your error. Circle or otherwise indicate what caused the error. Correct the error or work the problem over.

These rules are general guidelines and are designed to develop skills that will help you prepare for your tests. Some of these rules will be covered later in the supplement but a few comments are necessary at this time.

Rule 1-3,7: To be covered in detail later in the supplement.

Rule 4: It may appear that paper can be saved if you work across the page but this will lead to errors in your work and it makes the problems difficult to check for both you and your teacher. **WORK DOWN!**

Rule 5: It is far easier to find problems when they occur than to try to find them later. A small error in a step will cause the rest of the problem to be wrong. When this happens a simple problem may be made much more difficult and require a great deal more time. At each step check your work. If it appears correct put a check beside it. It might appear that it would take more time do this but in reality less time will be required. Checking is more effective if you do not try to do the step exactly the same way. For example if the step was to factor $x^2-3x-18$ yielding:

$$(x-6)(x+3)$$

It would be better to multiply the answer to see if it resulted in the original line than to try to factor the first line again. Sometimes, because of difficulty in reversing a step, you may need to check by doing the step over.

Check each step! If correct put a check beside it.

Rule 6: Many mistakes are caused by a loss of concentration midway through a problem. Give yourself enough time and don't look up until the problem is finished.

Rule 8: Remember, the daily work is designed to help you succeed on the tests. You need to work the daily problems in such a way that the test problems can be worked in the same manner as practiced on the homework. The difficulty is that usually the answers are available for the daily work but on tests you do not have them. In order to compensate for this you must do the daily work with maximum effort directed at getting the problem right the first time. In order to accomplish this I want you to put an X by any problem you get wrong on the first try, even though you subsequently correct it. Remember, using Rule #5 you are checking each step as you go, so many problems will be correct on the first try. If you get many problems wrong, in any set on the first try, you need to continue practicing until you get the problems right the first time. **Get them right the first time!**

Rule 9: Errors tend to reoccur. If you use the homework for preparing for tests and you have marked the type of errors that you have made, you can avoid repeating them.

SECTION 2: PROBLEM SOLVING IN ALGEBRA

We will now look at some general ideas on solving problems in algebra. Probably the most important reason for taking algebra is the problem solving skills that you should develop. When you encounter a problem in algebra there are certain steps you should take whether the problem occurs in daily work or on a test.

1. Categorize the problem.
2. Develop a plan.
3. Work the problem according to the plan.
4. Check your work.

The first step, categorize the problem, is the step that is the most difficult to get from the book. This is because the method that is most often used by students to solve problems is to look back in the book to see how it is done. This, coupled with the fact that the book groups problems that are alike in the same section, leaves the student (you) with little practice in categorizing the problem. You can overcome this problem by carefully studying not only how to solve the problems but also by paying close attention to the type of problem that is likely to occur. For example, the topic of fractions might include problems of the following types: reducing, multiplying, dividing, adding, solving equations, complex fractions and word problems. You may know how to solve equations and add fractions but if you use the plan for solving equations to add fractions the result will be poor indeed. Before you start any problem you must think carefully about what kind of a problem you are working. The instructions for the problem give the key to categorizing. It will never say solve if what is required is to add. *You* must look at these key words and associate the correct method with them. Some of these words will not have a clear meaning at first. "Simplify" is a common word that often confuses students. How simple is simple? When are you finished? The answer to these questions will be covered in the text but one thing is certain. Solve and simplify mean something different. Consider the following example:

Perform the indicated operation and simplify:

$\frac{1}{x-2}-\frac{2}{x-3}$ Perform the indicated operation, in this case, means subtract.

Simplify, in this case, means reduce the answer.

$(x-2)(x-3)\frac{1}{x-2}-(x-2)(x-3)\frac{2}{x-3}$ ✓ Multiply by LCD.

$x-3-2x+4=0$ ✓ Reduce

$-x+1=0$ ✓

$x=1$ Wrong answer! What happened here?

The instruction **simplify** was confused with the instruction **solve.** You find the value for x only if the instruction is solve. Do not add an = sign to a problem that does not have it.

Correct solution: $\frac{1}{x-2}-\frac{2}{x-3}$ Find LCD. $(x-2)(x-3)$

$\frac{x-3}{x-3}\cdot\frac{1}{x-2}-\frac{x-2}{x-2}\cdot\frac{2}{x-3}$ ✓ Convert each fraction to an equivalent one with the LCD.

$\frac{x-3-2x+4}{(x-3)(x-2)}$ ✓ Add the fractions.

$\frac{-x+1}{(x-2)(x-3)}$ ✓

No exact method of checking is available at this time but if time permits values could be chosen for x and substitutions made into the problem and answer to see if the value is changed. If it has then you have made an error in your work. If not you may have more confidence in your answer but it is still not proved.

Catergorize the problem. Develop a plan. Follow the plan!

GENERAL PROBLEM SOLVING TECHNIQUES

In spite of careful preparation, there will be times both on daily work and on tests that you will not be able to solve a problem. You draw a blank. What do you do? Of course you can get help from the Math Lab, a teacher or a friend and at times that is exactly what you should do, but that won't help on the tests. Remember, the skills you develop in your daily work will help you on the tests. You need to practice problem solving in your daily work. If you depend on outside help and problems being worked in class you will miss the necessary step in problem solving. **Work the problems yourself.**

There are some basic questions that you should use to improve your problem solving skills.

A. Understanding the problem.
1. What is the unknown?
2. What are the data?
3. What is the condition?
4. Is it possible to satisfy the condition?
5. Did you draw a figure?

B. Devising a plan.
1. Have you worked a similar problem before?
2. What connection is there between the data and the unknown?
3. Can you categorize the problem into one of the basic types?
4. Have you used all of the data?

C. Carrying out the plan.
1. Did you carry out the plan completely?
2. Did you check each step?

In algebra you will almost never be asked to solve some problem without seeing one of the same type before. You must identify the type and know how to solve it. The key words used in the problem will help you plan your solution. Still no idea? The best advice I can give is to do something. Simplify, add and reduce fractions, combine like terms and remove parentheses. In equations you can add, subtract, multiply, square (check for extraneous roots), take the square roots ($\pm$) of both sides. In word problems choose a letter or letters for the unknowns. **Try something!**

SECTION 3: PREPARING AND TAKING TESTS

The objective of the course is for you to learn algebra. The principal way that we measure this is through the tests in the course. This includes quizzes, unit tests and the final examination. Students can often be heard to say, "I can do the problems on the homework but I do not do well on the tests." It is the objective of this section to help with this problem. This will be done in two parts:

1. Preparing for tests.
2. Taking tests.

PREPARING FOR TESTS

If you have prepared your daily work carefully you should have the problem of preparing for any test ninety percent accomplished. This, of course, does not mean you would get 90 % on the test. With a 10% error factor you might miss all of the problems. In fact, I have observed that a few small hints given at the right time could improve student scores dramatically.
What you need to do is learn to provide your own hints.

The first thing you need to do is to review all types of problems that the test will cover. This should especially include problems that you worked without difficulty in the homework. These are your bread-and-butter problems and if you miss these "easy ones" your grade will be terrible.

Study the easy ones!

Work problems of each type until you get them right the first time. Check each step. Review your homework. Pay particular attention to the places that caused you to miss problems the first time. Look for the circled parts. This is where the time you took to indicate what caused you trouble in previous problems will pay off. Remember, on the test the answers will not be available. The habits you develop in your study will effect how you do the test.

Practice categorizing your problems. Use the practice tests in the book to do this. Before you start a problem think: what type of problem is it and what plan will work to find the solution. If you must go back into the text to do problems in the chapter tests you need more study.

Now you ready to take the practice test in this supplement. Before you start let me remind you that these tests are designed to find out if you are ready for the test and you should take it with this in mind. Give yourself an entire hour. Put your book away. Work each problem carefully. Remember to check each step. Leave any problem you cannot work. After you finish check the answers. Play it straight. Only you will know your score. Ask yourself the question, "Did I do well enough so that I am ready for the test?". If the answer is no then you need more preparation. Remember, the Math Lab and I will willingly help you with any type problem that you cannot work, but you must give yourself enough time to get it. Start your preparation early.

TAKING THE TEST

You are now prepared for the test. This does not assure a great grade. You must take the test in such a way that what you know is communicated to me. There are a few basic ideas that will help.

1. Work each problem just as you practiced. Categorize, plan and check.
2. Work the easy problems first. Don't hurry! Remember these are important to your grade. Leave room for any problem you skip.
3. If a problem is taking too long, leave it! Come back if time permits.
4. Show your work. If you are in doubt about whether a step should be included, put it in. Even if you miss the answer there may be some partial credit and these points count. Don't erase a problem and leave me nothing to grade.
5. Neatness counts. You need it to work the problems and I need it to grade accurately.

SECTION 4: CHOOSING AND USING YOUR CALCULATOR

A calculator will help you very little and could hurt you a lot in elementary algebra. You must learn to use it effectively and know when to use it, then it can be of some use in a limited number of problems. If you use it on tests and have not practiced with it, your grade will suffer in three important ways. First, you will take time from the real business of doing algebra. Second, you will miss answers because of doing the calculations wrong. Third, you will leave out steps that may be necessary in the grading. Still, because it can help students who use it correctly and knowing how to use it is a skill that can be valuable in many situations, I allow and encourage it's use.

There are two basic types of calculators that can be of use to you in algebra. The first is the type that uses what I will call adding machine logic and the second uses the logic we most often use in algebra. Consider the problem $2+3\cdot 5=?$. If you enter this problem just as it appears into a simple calculator using adding-machine logic you will get 25 for the answer but if you enter it into one that has algebraic logic you should get 17 as the answer. Try this with any calculators you may have or are considering buying. The reason for getting 17 with some calculators is they perform the order of operations in the manner that we do in algebra. That is, in this problem they multiply before they add. Although either type will work for you, in this course the type with the algebraic logic will be simpler to use. The other consideration might be price. Most calculators, especially those you carry with you on a daily basis, have a limited useful life, therefore: do not spend too much money.

Look now at a few types of calculator problems that might be useful in algebra. First let me say that the notation that we use in algebra does not always work well with your calculator. This is because many things in algebra are implied by the notation, whereas with your calculator everything must be entered.

For example, $3(4+5)=$ means multiply the sum of 4 and 5 by 3. The addition is done first as it would be on your calculator but the times operation is implied by the notation. To work this problem on your calculator you need to put in the multiplication operator. That is, you need to think of the problem as $3\cdot(4 + 5)=$. Put in the times operator.

The problem $\frac{3+4}{5+7}=$ uses the horizontal line to indicate both division and grouping. That is, you must think $(3+4)\div(5+7)=$ in order to use your calculator directly. This problem also brings out another problem with the use of calculators. The calculator gives an answer like .583333 and what we usually want is $\frac{7}{12}$. You must learn when decimal answers (approximate) or fractional answers (exact) are expected.

If you want to evaluate $x^2 - 3x + 4$ when $x = -2$, with your calculator, you may find it easier to use the memory function provided. In order to evaluate (key word), using an algebraic logic calculator, first enter 2 and then push the $\pm$ key to get -2. Push the M key to put -2 in memory. Then calculate the expression using the following steps:

Square, $-$, 3, $\times$, recall M, $+$, 4, $=$. You did not need to recall first because the -2 was already in your calculator register.

There are many types of calculators. In order to use yours consult your manual or see me. <u>If you do not practice with your calculator leave it at home on test day</u>.

CALCULATOR PRACTICE PROBLEMS

1. $3\bullet7.25+8\bullet1.25=$

2. $\frac{5.25-3.75^2}{4.3^2+2(1-3.75)}=$

3. $x=-2.4$ Find $x^2-3.4x-6.42$

4. $y=3x+5$ Find y when $x=-3.7$

5. -34.27^2

6. $(-1.234)^2$

7. $\sqrt{2.5^2-1.25^2}$

8. $\left(\sqrt{3.4}+\sqrt{5.6}\right)^2$

9. $\dfrac{\log(2)}{\log(3)+\log(5)}$

10. $\dfrac{-3-\sqrt{(-3)^2-4(2)(-3)}}{2(2)}$

SECTION 5: AN ALTERNATE METHOD OF SOLVING WORD PROBLEMS

Many students have trouble solving word problems. This type problem is especially important if you have to apply mathematics in the solution of a problem. Any real problem that you can describe in words becomes a word problem. If you cannot solve them you cannot apply algebra. If you cannot apply algebra it will do you little good.

The method we will examine here will be illustrated by the following example.

If four times the smallest of three consecutive integers is added to the largest the result is 112. Find the integers.

Solution: We will start this method by guessing the answer. Then we will try to check to see if the guess is correct. We will try to guess as little as possible and remember that we are not trying to guess the answer but to write an equation that will lead to the solution. Let's try 25 for the first number. Is this correct? Try it. ··· I get 127. Four times the first plus the third: $4(25)+27=127$. Most of us can tell whether the answer is correct even if we cannot work the problem. How can this be used to solve the problem? Let's guess again: 23. Wait, before you check to see if it is correct, write the other numbers in terms of the first. That is, instead of 23, 24, 25 write 23, $23+1$, $23+2$. Wait to do the arithmetic. Now check using this form.

$4(23)+(23+2) \stackrel{?}{=} 112$ #1 Now do the arithmetic.

$4(23)+25=$ ✓

$92+25 =$ ✓

$117 \neq 112$. Wrong answer but then we were not trying to guess the answer. Now guess again, but wait. Guess x. Then what are the numbers x, x+1, x+2. The check: $4x+(x+2) = 112$ #2

$4x+x+2=112$ ✓

$5x+2 = 112$ ✓

$5x+2-2 =112-2$ ✓

$5x=110$ ✓

$x=22$, $x+1=23$, $x+2 =24$ and $4(22)+24=$

$88+24 =112$ ✓

Look at the lines numbered #1 and #2. We put x in exactly like we used the numbers. For more information on this method check with the Math Lab for the video tape: An Alternate Method of Solving Word Problems.

A FURTHER NOTE ON WORD PROBLEMS

In algebra there are only a few main types of word problems. Regardless of the method you choose to use in solving them, there is one more important fact to remember. You need to work on one type at a time until you know how to do that type. For instance, there may be seven rate problems in a particular chapter in the book. Only three may be assigned but you may need to work them all if that is what it takes to develop your skills in that type problem. When word problems appear in a later chapter you will be able to work rate problems and then you will be able to concentrate on some other type of problem.

Remember, focus on one type of problem until you have it mastered.

SECTION 6: WHERE YOU CAN GET HELP

Where do you go when you need help and when do you need it? Generally on a day to day basis most of your help will come from the text. Every type of problem that you will be expected to work is covered there, both in theory and by example. Answers to the odd numbered problems are in the appendix. You need to work with the text until you are familiar with it. Your first source of material will be your text. Assignments will be made from problems chosen from it. After you have attended the lecture and read the book you should be ready to start the homework. Don't skip a step. Many students try to work the homework without adequate background by following the examples in the text, but this will not be enough information for you to work the problems effectively.

Start any assignment slowly. Remember, not enough problems may be assigned for you to catch on to an idea. *If you are having difficulty with a problem go back and work some more easy ones.* Look for the step that is causing the trouble. If you cannot find the trouble then go on. Don't bog down. You need to try all of the problems assigned before the next class. If you have a significant number of problems you are not able to solve, you need help. Don't leave them for the teacher. You need to solve them yourself. Seeing them on the board will reinforce them but if you have not worked them before they will be reduced to just another example. You need to spend two hours outside of class for each hour in class. If the homework is taking significantly more time than this something is wrong.

You need help. Where do you get it? Three main resources are available here at the college. First see your teacher, that's me. If you need help come and see me first. I keep regular office hours every week and if these do not fit your schedule I will arrange others. I want to help you. Give me the first chance. Come to me early and often. I will be able to look at your work and decide what is going wrong better than anyone else. Working problems in my office can get you going. Don't wait for a bad test result. My office is 239E. The regular office hours are posted on my door and announced in class.

If you need more than this we also have the Math Lab in 7-126. This facility is open every day from 8 a.m. to 10 p.m. except Friday when it is open from 8 a.m. to 5 p.m.. Some students find that working the majority of their homework there is about the only way to accomplish it in the two hours allotted.

In the math lab the most important resource is the tutors. Please remember that they are not trained teachers. Don't expect them to do either of our jobs. They are not supposed to work the problems for you. You ask: "Where am I going wrong?". Not: "Work this problem for me". If new methods are presented to you in the lab ask the tutor to do it by the methods of the book. I use this method, you use it. They can help you with it. If not, see another tutor or see the person in charge of the lab. She is trained and can give you first class help.

The best hours for the lab are in the afternoon or evening when there is a crush of students. If you need more than 5 or 10 minutes help an hour you are not using the lab correctly. You need to work the problems yourself. Also available in the lab are video tapes on the topics of algebra. These may be useful to supplement the lecture.

Tutors are also available in the study skill area of the library. As in the lab these are good students that can help you if you give them a chance. Shop around early. Find a tutor you can work with before some crisis exists. Don't wait until the test. By then it may be too late.

SECTION 7: SOLVING INEQUALITIES GRAPHICALLY

Inequalities are among the most important tools you will learn in algebra. In this section we will consider a somewhat different approach from that of the text. This method will be useful principally in inequalities that have the form:

$\frac{f_1(x)f_2\cdots f_n(x)}{g_1(x)g_2(x)\cdots g_m(x)} < 0$. That is, both the numerator and denominator are factorable into a form where the values, if any exist, that make the factors zero can be found. We will illustrate the method with a simple example.

Example: Solve $\frac{(x-3)(x+2)}{(x-1)} < 0$

In this problem it is apparent that the zeros are +3, −2, and +1. We will put these numbers on the number line. This is exactly what we would do using the method of the book. Then we will choose some other number, not a value that makes a factor zero, and compute whether it makes the left side of our inequality + or −. In the example, if we pick $x=4$ all three factors are +, thus their product is +. At some point above $x=4$ locate a point. Now we can draw the schematic curve. Starting at the point we have at $x=4$, crossing at +3, +1 and −2.

SECTION 8: WHAT TO DO IF YOU THINK YOU SHOULD DROP

Sometimes in any course it may appear to you that you should drop. There are certain things you should do before you do this. First see your instructor. You may not be as bad off as you appear or there may be something that we can do to help the situation. **Come see me!**

Students often make the decision to drop unconsciously. It often starts with the decision to miss class because you are not prepared. This is often the time when coming to class is the most necessary. You will just get further behind. Get help! Go to the Math Lab. Since you have invested this much time do not drop without careful thought.

If you decide to drop take note. In the first five weeks you may be able to drop back to an easier course. This will not help if you are not willing to do the work but if you are trying to do well and can't, see me.

Drop back, don't drop! State law prohibits dropping after the thirteenth week. If you stop attending after that week you will fail the course.

No drops after the thirteenth week! Take the final. Things may be better than they seem.

SECTION 9: NOTES ON TESTS

NOTES ON TEST 1:

The first test is important as it can set a trend for the entire course. You may have heard the expression "math anxiety". This describes a Math D student who does poorly on the first test. You may have taken Intermediate Algebra sometime in the past and be in a position where you "know this stuff" and do not need to study. You must be especially careful as some small things can trip you up at this early stage. Problems like -2^2 can cause difficulty if you do not consider the order of operations and do not do the squaring first. The problem $3+2(4+x)$ is of the same type. You must remove the parentheses first. You cannot add the three and the two first. In more complicated problems simplify by removing the innermost parentheses first.

Example: Simplify $3+5[\{2-4x\}-2x]$ Think what does simplify mean?

$3+5[2-4x-2x]$ ✓

$3+5(2-6x)$ ✓

$3+10-30x$ ✓

$13-30x$ ✓ Notice we did not find x.

Example: Solve $2-4[1-3x]=-5$

$2-4+12x=-5$ ✓ Get the signs right!

$-2+12x=-5$ ✓

$-2+12x+2=-5+2$ ✓ Add two to both sides.

$12x=-3$ ✓

$x=-\frac{1}{4}$ ✓ Solving we found the value of x.

Check: $2-4[1-3(-\frac{1}{4})]=$

$2-4[1+\frac{3}{4}]$

$2-4[\frac{7}{4}]$

$2-7=-5$

NOTES ON TEST 2:

Example: Add and simplify

$$\frac{1-x}{x^2-x}+\frac{2x}{1-x^2}-\frac{2-x}{x}$$ First find the LCD.

$$\frac{1-x}{x(x-1)}+\frac{2x}{(1-x)(1+x)}-\frac{2-x}{x}\ \checkmark$$ Reduce.

$$\frac{-1}{x}+\frac{2x}{(1-x)(1+x)}-\frac{2-x}{x}$$ Find LCD. $(1-x)(1+x)\cdot x$ ✓

$$\frac{-1(1+x)(1-x)}{x\cdot(1+x)(1-x)}+\frac{2x\cdot x}{(1-x)(1+x)\cdot x}-\frac{(2-x)(1-x)(1+x)}{x\,(1-x)(1+x)}\ \checkmark$$

$$\frac{-1+x^2+2x^2-2+2x^2+x-x^3}{x(1-x)(1+x)}\ \checkmark$$

$$\frac{-x^3+x^2+x-3}{x(1-x)(1+x)}$$ ✓Keep the denominators. This is not an equation.

Example: Solve

$$\frac{1}{x-2}+\frac{2}{x+1}=\frac{1}{2}$$ Again find the LCD but this time multiply each side by it and reduce.

$$2(x-2)(x+1)\frac{1}{x-2}+2(x-2)(x+1)\frac{2}{x+1}=\frac{1}{2}2(x-2)(x+1)\ \checkmark$$

$$+2x+2+4x-8=x^2-x-2\ \checkmark$$

$$x^2-7x-8=0\ \checkmark$$

$$x=\frac{7\pm\sqrt{33}}{2}\ \checkmark$$ Here the solution was obtained using the quadratic formula.

Example: Solve

$\sqrt{X-2}-\sqrt{X+3}=3$ REMEMBER $(A+B)^2 \neq A^2+B^2$

$\sqrt{X-2}=\sqrt{X+3}+3$ ✓Now square both sides. *The left side is a binomial.*

$X-2=X+3+6\sqrt{X+3}+9$ ✓ Isolate the radical and simplify.

$-14=6\sqrt{X+3}$ ✓ Divide each side by 2.

$-7=3\sqrt{X+3}$ ✓ Square each side again.

$49=9(X+3)$ ✓

$X=\frac{22}{9}$ ✓ Now check.

Remember when you square an equation you may add numbers to the solution set that will not work in the equation.

Check:

$$\sqrt{\frac{22}{9}-2}-\sqrt{\frac{22}{9}+3}=3$$

$$\sqrt{\frac{4}{9}}-\sqrt{\frac{49}{9}}=3$$

$\frac{2}{3}-\frac{7}{3}\neq 3$ Therefore there is no solution.

Example: Given $f(x) = x - x^2$ Find $\frac{f(x+h)-f(x)}{h}$

Note: $f(x+h)$ means substitute $x+h$ into $f(x)$ for x. $f(x+h) \neq f(x)+h$.

Solution:

$$\frac{(x+h)-(x+h)^2 - (x-x^2)}{h} \quad \checkmark$$

$$\frac{x+h-x^2-2xh-h^2-x+x^2}{h} \quad \checkmark \text{ Watch the signs!}$$

$$\frac{h-2xh-h^2}{h} \quad \checkmark \text{ Now reduce the expression.}$$

$$1-2x-h \quad \checkmark$$

NOTES ON TEST 3:

It is very important that you be able to recognize the type of curve by examining the equation. Although this is not always possible for many of the equations in this section it can be easily accomplished. The equation must be of the type $AX^2+BY^2+CX+DY+F=0$ then it can be identified by the following criteria. (A and B not both zero.)

1. If $A=0$ OR $B=0$ then the equation represents a parabola.
2. If $A=B$ then the equation is a circle.
3. If $AB>0$ and $A \neq B$ then the equation is an ellipse.
4. If $AB<0$ then the equation is a hyperbola.

Note: The curves may also be degenerate cases. For instance $x^2+y^2+4=0$ tests as a circle since $A=1$ and $B=1$ therefore $A=B$, but it has no points that satisfy it with x and y real numbers. $x^2-y^2=0$ appears to be a hyperbola with $A=1$ and $B=-1$ therefore $AB<0$, but it is actually two straight lines. When you test an equation it will never test a circle when it's a hyperbola but it may be lines, points or nothing!

Example: $2X^2 - 3Y^2 + 4X - 3Y + 4 = 0$ tests to be a hyperbola since $A=2$ and $B=-3$ therefore $AB < 0$.

Remember to put the equation in standard form before applying tests.

A special type of graphing problem can be illustrated by the following:
Example: Graph $X^2 - 3xy + 2x = 0$ The tests from the preceding example do not work because of the xy term but because of the common factor we can use another method. First factor the left side, yielding

$x(x - 3y + 2) = 0$. Now each factor is set to 0.

$x = 0$ and $x - 3y + 2 = 0$ (you can graph these).

The graph of these two straight lines is the graph of the original equation. This is based on the property that if $ab=0$ then $a=0$ or $b=0$. One side must be 0 and the other side factorable in order for this to work.

Example: Graph $(x^2 + y^2 - 4)(x^2 - y^2 - 4) = 0$

Since the left side is factored and the right side is zero the equation can be graphed by graphing $(x^2 + y^2 - 4) = 0$ and $(x^2 - y^2 - 4) = 0$. This is left as an exercise.

The material on logarithms is fairly complete in the text but if you feel you need more review there is a video tape in the Math Lab that may be of help. One of the problem areas in logs occurs in problems of the following type:

Example: Use the properties of logs to write as a sum.

$$\log_b \frac{x^2(x-y)}{yz}$$

$$2\log_b x + \text{lob}_b(x-y) - \text{lob}_b(y) - \log_b(z)$$

Note that $\log_b(x-y)$ cannot be simplified and since we are dividing by z it must be subtracted.

Mathematics D SAMPLE TEST-SHOW YOUR WORK Name______________

Examination 1 USE SEPARATE PAPER Section______________

1. a, c, d

1. Let A = {a, b, c}, B = {a, b, c, d}, and C = {a, c, d}. List the elements in the set $(A \cap \varnothing) \cup (B \cap C)$.

2. $1, \frac{2}{5}, \frac{0}{8}$

2. List the numbers in the set, $\left\{1, \frac{2}{5}, \frac{0}{8}, \sqrt{7}\right\}$, that are rational numbers.

3. comm.prop. of add.

3. Give the property of equality or the property of real numbers that justifies the following statement: $3 + (x + 2) = (x + 2) + 3$.

4. $-\frac{1}{3}$

4. Find the additive inverse of the reciprocal of 3.

5. 20

5. Assume that $x = -50$, $y = 10$, and $z = -5$. Evaluate the expression. Simplify the result, if necessary.

$$\frac{3xy + 2yz}{2(x + y)}$$

6. 8

6. Simplify the expression: $|4 - 2| + |8 - 5| \div |3 - 6| \bullet |2 - 8|$.

7. $\frac{4y^4}{25}$

7. Simplify $\left(\frac{2y^{-1}}{5y^{-3}}\right)^2$. Write the answer **without using negative exponents**.

8. −8 10

8. Use synthetic division to find the quotient when $4y^3 + 3y - 1$ is divided by $y + 2$.

9. $x^{-n}(x^{3n} - x^{2n})$

9. Factor x^{-n} from $x^{2n} - x^n$.

10. $3(a-3)(a^2+3a+9)$

10. Factor completely: $3a^3 - 81$.

11. $(3a - 2b(2a + 5b)$

11. Factor completely: $6a^2 + 11ab - 10b^2$.

12. $(r-s-4)(r-s-3)$

12. Factor completely: $(r - s)^2 - 7(r - s) + 12$.

13. $(x^n + 3)(x^n - 5)$

13. Factor completely: $x^{2n} - 2x^n - 15$.

14. $(x+3)(y+2)(y-2)$

14. Factor completely: $xy^2 - 4x + 3y^2 - 12$.

15. $(x+3+y)(x+3-y)$

15. Factor completely: $x^2 - y^2 + 6x + 9$.

16. 3

16. Solve: $\frac{2 + x}{3} + \frac{2x + 1}{6} = \frac{2x + 11}{6}$

17. 11 cm

17. A rectangle is 5 centimeters longer than it is wide, and its perimeter is 34 centimeters. What is the length of the rectangle?

18. 8 inches

18. Weights of 5 and 8 pounds are placed at opposite ends of a steel bar that is 13 inches long. How far from the fulcrum should the 5-pound weight be placed so the weights are in balance?

19. 4 mph

19. A man bicycled for 3 hours and walked for 1 hour to cover a distance of 40 miles. He rode three times as fast as he walked. How fast did he walk? (Assume that he did not stop to rest.)

20. $n = \frac{-360}{a-180}$ or $n = \frac{360}{180-a}$

20. Solve for n: $a = \frac{180(n - 2)}{n}$

21. 0, 1, -5

21. Solve for x: $x^3 + 4x^2 - 5x = 0$.

22. 6, -1

22. Solve the equation. It has two solutions.
$|3x - 4| = |x + 8|$

23.

23. Solve the inequality. Graph the result on a number line.
$5 \leq 2x + 1 \leq 9$

24. 

24. Solve the inequality. Graph the result on a number line.
$|x + 1| \geq 2$

Mathematics D SAMPLE TEST-SHOW YOUR WORK Name____________________

Examination 2 USE SEPARATE PAPER Section________________

Answer	Problem
1. $x + 2$	1. Simplify: $\dfrac{2x^3 + 16}{2x^2 - 4x + 8}$
2. $\dfrac{c + d}{c + 1}$	2. Simplify: $\dfrac{ac + ad + bc + bd}{ac + a + bc + b}$
3. $\dfrac{x}{x + 1}$	3. Perform the indicated operation and simplify. $\dfrac{x^2 + x - 6}{x^2 + 4x + 3} \bullet \dfrac{x^2 + x}{x^2 - x - 2}$
4. $\dfrac{x - 5}{x(x - 25)}$	4. Perform the indicated operations and simplify. $\dfrac{x^2 + 5x + 4}{x^2 - 16} \div \left[\dfrac{x^2 + 4x + 3}{x^2 - 5x} \bullet \dfrac{x^3 - 25x^2}{x^2 - x - 12}\right]$
5. $\dfrac{-x^2 + 15x + 18}{6(x + 3)(x - 3)}$	5. Perform the indicated operations and simplify, if possible. $\dfrac{3}{x^2 - 9} + \dfrac{x}{3x - 9} - \dfrac{x}{2x + 6}$
6. $\dfrac{1}{y - x}$	6. Simplify: $\dfrac{\frac{1}{xy}}{\frac{1}{x} - \frac{1}{y}}$
7. $\dfrac{xy^2 - x^2}{y^2 + x^2y}$	7. Simplify: $\dfrac{x^{-1} - y^{-2}}{x^{-2} + y^{-1}}$
8. 5	8. Solve: $\dfrac{5}{y + 4} + \dfrac{2}{y + 2} = \dfrac{6}{y + 2} - \dfrac{1}{y^2 + 6y + 8}$
9. 4 mph	9. In a river that has a current of 2 miles per hour, a Boy Scout can row 12 miles downstream in the same amount of time that he can row 4 miles upstream. How fast can the boy row in still water?

10. 3

10. Write the expression $\frac{a^{-3/2}b^{1/3}c^{-2}}{a^{1/2}b^{-8/3}c^{-4}}$ in the form $a^x b^y c^z$. Find x + y + z.

11. $-\frac{\sqrt{2z}}{2z}$

11. Simplify: $\frac{\sqrt{200z}}{4z} - \frac{\sqrt{98z}}{2z} + \frac{\sqrt{288z^3}}{24z^2}$ (Assume x > 0.)

12. $\frac{a - 4\sqrt{ab} + 4b}{a - 4b}$

12. Rationalize the denominator and simplify:

$$\frac{\sqrt{a} - 2\sqrt{b}}{\sqrt{a} + 2\sqrt{b}}$$

13. $\frac{(z - 2)\sqrt{z^2 - 7}}{z^2 - 7}$

13. Rationalize the denominator and simplify:

$$\frac{\sqrt{z^2 - 7} + \frac{3}{\sqrt{z^2 - 7}}}{z + 2}$$

14. -2

14. Solve for x: $\sqrt{x^2 + 5} = 5 + x$

15. $\frac{5}{4}$

15. Find the slope of the line that passes through the points (0, 0) and (4, 5).

16. $\frac{2}{3}$

16. The graph of the equation $-2x + 3y = 14$ is a straight line. Find its slope.

17. $y = -\frac{4}{3}x - 1$

17. Write the equation of the line passing through (−3, 3) and (3, −5).
Write the answer in slope-intercept form.

18. perpendicular

18. Indicate if the graphs of the equations

$$\begin{cases} 3x + 4y = 12 \\ 2x = 3\left(\frac{y}{2} + 2\right) \end{cases}$$

are parallel, perpendicular, or neither.

19. all real num. excp. 3 19. Assume that $f(x) = \frac{x + 4}{x - 3}$. Find the domain of f.

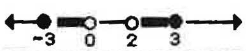
20. $3s^2 - 8s + 9$ 20. Assume that $f(x) = 3x^2 - 2x + 4$. Find $f(s - 1)$.

21. $f^{-1}(x) = -5x + 15$ 21. Find the inverse relation of $x + 5y = 15$. Write the answer in the form $f^{-1}(x) = mx + b$.

22. $f^{-1}(x) = \frac{3x + 3}{x}$ 22. Find the inverse relation of $y = \frac{3}{x - 3}$. Write the answer in the form $f^{-1}(x) = mx + b$.

23. 4 hours 23. For a fixed distance d the rate of speed varies inversely with time. If the time is 5 hours when the rate is 48 mph, find the time when the rate is 60 mph.

Mathematics D	TEST-SHOW YOUR WORK	Name________________
Examination 3	USE SEPARATE PAPER	Section______________

Answer	Question
1. $\frac{-3+\sqrt{13}}{2}, \frac{-3-\sqrt{13}}{2}$	1. Use completing the square to solve the equation: $y^2 + 3y - 1 = 0$
2. $9 - 7i$	2. Multiply: $(3 + \sqrt{-1})(2 - \sqrt{-9})$. Write answer in $a + bi$ form.
3. 2, −2, 4, −4	3. Solve the equation. $y^4 - 20y^2 + 64 = 0$
4. 16	4. Solve the equation. $y - y^{1/2} - 12 = 0$
5.	5. Graph the equation $y = 2x^2 - 4x - 3$. Find the vertex.
6. 15, 15	6. Find two numbers whose sum is 30 and whose product is maximum. Show your equation and solve.
7. 	7. Solve the inequality and graph its solution set on a number line. $\frac{2x^1 - 9}{x^2 - 2x} \le -1$
8. (2, 5)	8. Find the coordinates of the center of the circle $x^2 - 4x + y^2 - 10y + 17 = 0$. Graph.
9. 2	9. Consider the system $\begin{cases} x - y = -3 \\ 2x + 3y = 4 \end{cases}$ which is to be solved by using Cramer's Rule. Solve the system for y.
10. 3	10. Solve for y. $\begin{cases} y + z = 4 \\ x + z = 1 \\ x + 4y = 12 \end{cases}$

11.

11. Solve the following system of inequalities by graphing.

$$\begin{cases} x^2+4y^2 \le 4 \\ x^2-9y^2 > 9 \end{cases}$$

12.

12. Carefully graph $y = 2^{-x}$.

13. $3\log_b x - 2\log_b y$

13. Write the expression in terms of the logarithms of x and y:

$$\log_b \frac{x^3+1}{y^2}$$

14. $\log_b \frac{x^2}{y^4}$

14. Write the expression as the logarithm of a single quantity:

$$2\log_b x - 4\log_b y$$

15. $x = \frac{\log 3}{1 - \log 3}$

15. Solve for x: $3^{x+1} = 10^x$. **Do not replace logarithms with numerical values.**

16. $x = 1$

16. Solve for x: $\log x + \log(x + 9) = 1$

17. 13

17. Evaluate $\begin{vmatrix} 2 & 0 & 1 \\ -1 & 2 & 1 \\ 2 & -3 & 2 \end{vmatrix}$.

18. $y = \frac{1 + 2x}{x}$

18. Find the inverse relation of $f(x) = \frac{1}{x - 2}$. Is $f^{-1}(x)$ a function? Why?

19. 64

19. Solve the equation. **Do not list any extraneous solutions.**

$$2x^{1/3} - 3x^{1/6} - 2 = 0$$

19. Graph the equation $|2x| - y = 2$.