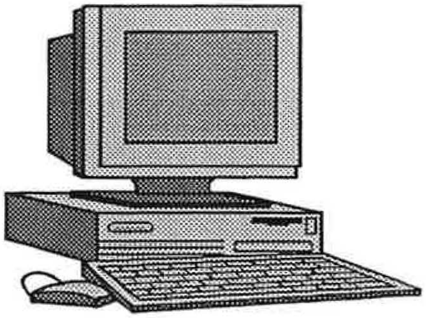

CLASSROOM PRESENTATIONS

USING MULTI-MEDIA

Sabbatical Report--Spring Semester, 1999

Mary Chabot

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Applicant: **Mary Ann Chabot**

PROPOSAL FOR SABBATICAL PROJECT

Feedback from my students and their subsequent success has convinced me that student learning is increased when the traditional lecture format is supplemented by student projects and classroom presentations utilizing technology. This seems to be especially true when these projects or presentations provide visual images of the topic under discussion.

These projects and presentations also excite and motivate a group of students who, in the past, were not afforded this additional avenue of understanding, and were not able to "see" the visual aspect of the concept on their own.

Currently, I have been able to bring myself up to speed with graphing calculator technology, through attendance at math conferences, networking with colleagues, and studying on my own. I have also, through staff development classes, attained a familiarity with the computer algebra system called DERIVE. However, the images that these afford are primitive by present day standards.

I am requesting a sabbatical leave in Spring of 1999 to make myself current in the world of multi-media and develop at least 4 student projects or classroom demonstrations that involve multi-media for the precalculus/calculus sequence. I am proposing at least 4 projects because I do not have access to the appropriate computer equipment at home, and also because I do not know the exact time needed to truly learn the authoring programs and the time needed to develop a single project. At this point I am valuing quality over quantity.

Two other members of my department have received grants to develop multi-media, among other things, in the business calculus class (Math 140). Through one of the grants the college has acquired a MAC lab with computers capable of performing multi-media projects, and these computers are accessible to other students as well. This affords us the opportunity to offer precalculus/calculus students this alternative method of learning.

My present knowledge of multi-media is next to nil. Therefore, I plan to get a "head start" by familiarizing myself over the next year, as much as my class schedule will permit, with multi-media authoring programs, as well as other instructional areas on campus and other institutions which are utilizing them in classes.

Applicant: **Mary Ann Chabot**

Tentative Time-line:

January, February:

I will spend the first two months becoming expert with the technology, and targeting the concept I wish to develop in the first presentation/project.

On campus, I will be contacting Dwight Ayle, Wayne Lutz, Jesse Mezquita, Terri Beam, Joan Sholars, Kevin McDonald, and Charles Sorcabal, all of whom I know at this point have experience with multi-media development. I will also contact other on-campus instructional areas, since I plan on researching who else might be of assistance to me. I will ask them for any advice on development, any pitfalls they can help me avoid, and their opinions on what has been most valuable for the students' learning.

Off campus, I will contact other institutions who are using multi-media in the math classroom to observe their demonstrations and discuss the successes/failures they have experienced. I will be contacting CMC^3 - South, our local math professional organization, for a list of math instructors who are involved in multi-media classroom use.

I will also be consulting the Internet to research what is currently being done for precalculus/calculus using multi-media instructional modes.

March, April, May:

Develop at least four projects/demonstrations. At this point, I am unable to provide further detail. The reason for this is two-fold. First, my knowledge of the medium is next to nil, and I am requesting a sabbatical to totally familiarize myself with this communication tool. Second, because the field is changing so rapidly, what I envision today may be able to be vastly improved upon by the technological innovations that exist a year from now.

However, some of the ideas that I have considered at this point are incorporating pictures or film of some of the applications that we deal with analytically. An example would be film of an actual chemistry lab experiment tied to the discussion of a differential equation that relates to chemical reactions. Another idea would be to provide visual images of fractals being developed, tied to the discussion of infinite series to find perimeters and areas of such objects. However, at this point these are just ideas, and my projects may go in other directions.

Applicant: Mary Ann Chabot

ANTICIPATED VALUE AND BENEFIT

This sabbatical will enable me to offer students an avenue of learning that, without the sabbatical leave, I would be unable to develop. However, it is not only the technology being introduced that I will be scrutinizing. By deciding which concepts are best demonstrated or enhanced by the multi-media projects/demonstrations, I will be re-examining the entire curriculum for the precalculus/calculus sequence, and keeping my ideas and my lectures fresh and new. In addition, I have found that any time I attempt to learn an area that is new to my understanding, I have a closer connection with the students' struggle with new material, and am more understanding as an instructor. So there will be a three-fold benefit to me: the ability to offer students an alternative method of delivery, the critical overview of the entire precalculus/calculus sequence, and the personal side of the instructor-student relationship.

The benefits to my department/division are also multi-faceted. First and foremost, the students will benefit from exposure to a visual understanding not otherwise available to them. In addition, they will be exposed to the power of current day technology, a knowledge of which is necessary for success in most work situations. An experience with technology widens their scope and enhances their abilities to utilize the technologies they will encounter in their future jobs. I will share my projects/demonstrations with my colleagues (thus benefitting all students in those classes, not just my own). I will be a resource to others who are desiring to incorporate multi-media into their classes, not only in my department, but also in the division and the college.

The College will benefit primarily because the students will benefit. In addition, this utilization of current technology and alternative method of instructional delivery will demonstrate concrete progress by the institution in fulfilling its mission/vision statements. There is also the possibility that I will be invited to present my innovations at conferences, thus representing the college in a very positive light to other institutions.

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STATEMENT OF PURPOSE

The purpose of this sabbatical was to familiarize myself with multimedia software and how that software could be utilized in the classroom to enhance student learning. In addition, it was to provide me with the time needed to develop at least four classroom presentations for the precalculus/calculus sequence.

Included in this report are the actual presentations I developed, as well as descriptions of the objectives accomplished by each individual project. Also submitted with this report are copies of each of the presentations on disk.

The work I completed during the Spring, 1999 semester and the knowledge I gained during that time will be directly applied in the classroom to our students. An expanded description of the value of my work to the college can be found in the summary of my conclusions, at the end of this report.

OVERVIEW

During the first two months of my sabbatical, I familiarized myself with a variety of multi-media authoring programs, including Director, Corel Presentations, and PowerPoint. I opted to use PowerPoint for three reasons. First, PowerPoint is installed on the computers we use in the classroom in the mathematics area. Second, it appeared to be a viable tool for accomplishing what I intended as far as classroom presentations. Last, it is a "standard" software program that is widely used, and thus provides a good basis for being able to extend myself to other programs. For example, when I contacted Dwight Ayle, he explained that FrontPage was a good tool for writing on the web, and that using FrontPage was very similar to using PowerPoint.

In addition, during January and February, I learned how to use MAPLE, one of the computer algebra systems that we have installed on the computers in the student math lab. I wrote some programs in MAPLE to animate graphics, and planned on using MAPLE and PowerPoint to develop the classroom projects.

I visited with Tom Vela, who had developed some PowerPoint lectures for his classes, and he shared some of his experiences with me. I also visited with Terri Beam, who gave me some insight into what was working and what was not working in her development of an on-line course for chemistry. Chuck Sorcabal, Kevin McDonald, and Scott Guth, in the mathematics area, were of invaluable assistance when I ran into troubles, either with PowerPoint or MAPLE.

In March, I started the actual writing. The first project I developed was "Optimization: The Box Problem". What I soon learned is that the animated gif.'s I was writing in MAPLE were not able to be imported to PowerPoint 97. (This problem is supposed to be rectified in PowerPoint 2000, which was not available when I was writing). I contacted several people on campus as well as the MAPLE representative at the Spring Conference of California Council of Mathematics-Community Colleges, and none was able to either import the animated gif.'s or successfully convert them to a form that PowerPoint 97 would accept. So I had to import the graphics frame by frame to achieve the animations needed for student visualization.

As a consequence I also created a MAPLE worksheet to go with one of the classroom projects. This is the idea I incorporated into "Several Variables: A Visual Overview", which is the fourth project I developed. Students can view the PowerPoint presentation, but then also go directly to MAPLE, and view the animations which are interactive.

In talking with colleagues at other schools, I learned that I am exploring fairly new territory with my projects. I also examined existing software packages developed by book publishers and available for student use with the adopted text, either on CD-ROM or on the web. Some are pretty fancy from a "wow" perspective, but are very short in the area of pedagogy and the educational objectives they are trying to accomplish. They are flashy but shallow. For each PowerPoint and MAPLE presentation, I carefully outlined the key ideas I was trying to convey, and how I could achieve my goal of enabling the students to visualize these concepts using multi-media. A detailed summary of these activities as they relate to the individual presentations is included in the body of this report.

PRESENTATION 1

EXPONENTIAL FUNCTIONS

FINANCIAL APPLICATIONS

This presentation is intended for use in a Precalculus class, but is also suitable for a course in Finite Mathematics. The portion on compound interest is also appropriate for the Intermediate Algebra student.

The presentation is intended to accomplish the following:

1) Provide real-life applications of mathematics, in particular exponential functions. Students often comment that they don't see any relevant usage for the mathematics they study in the classroom, and so I chose compound interest, annuities, and amortized loans. These are consumer-oriented applications, at least one of which they will find meaningful. The problem with these topics is that, in an ordinary classroom setting, the computations become very tedious, and the students are so busy with the calculations that they lose track of the overall points being made. With the PowerPoint presentation, they can view the slides and take notes on the accompanying handouts that this

program generates. They can concentrate on the concepts, rather than getting bogged down in the arithmetic. I chose to do the graphs and tables in EXCEL, because the finished product from this program looks similar to the types of real-life graphs students encounter in newspapers and magazines, rather than the graphs they see in a math class.

2) Provide the student with a visualization of how an exponential function differs from, for example, a linear function. With the latter, the growth in funds over any 5 year period will always be the same, while with the exponential function the growth in funds increases considerably over time. This is illustrated by the annuity example, where the growth in funds from years 35 to 40 is five times the growth from years 15 to 20.

3) Present the concepts numerically (tables), algebraically (formulas), and graphically (graphs). The mathematics reform movement has advocated that students see the material in a variety of ways, and most of the mathematical community has adopted this as pedagogically sound. However, solving a problem using such diverse methods gets to be very time-consuming when done at the chalkboard. The student often loses

interest once the problem has been solved one way, and that student's attention drifts while the other ways are being employed. The multi-media capabilities of PowerPoint with EXCEL enable the student to do all three methods almost simultaneously, thus exposing that student to a variety of ways to view the same problem.

4) Relate two mathematical topics that the students don't necessarily see as related; namely geometric sequences and exponential functions. These topics are not contiguous in most texts, and the students often see separate chapters in a text as totally unrelated to each other. At the precalculus level, one of the objectives is to have the students demonstrate synthesis and analysis of material. To visualize the graph of a geometric sequence as exponential in shape, or to relate the formula for the sum of terms in a geometric sequence to the exponential equation for the future value of an annuity is again very time-consuming using only a chalkboard.

5) Provide the student with an introduction to the capabilities of a spreadsheet program like EXCEL.

PRESENTATION 2

OPTIMIZATION

THE BOX PROBLEM

This presentation is suitable for a precalculus class or a first semester calculus class.

The presentation is intended to accomplish the following:

1) Provide a visual overview of this "classic" precalculus/calculus problem. Geometrical problems like these are often employed as applications, because it is felt that they are more concrete and physically accessible than some of the other applications. However, many students (especially at the precalculus level) are not able to grasp what is being asked in the problem. Many students can't visualize that an 8 inch by 10 inch piece of cardboard can produce very different looking boxes with different volumes. In addition, the student has great difficulty seeing how the algebraic formula for volume is derived. I have tried to go through this step-by-step in this presentation. The detail provided in the presentation is very time-consuming at the chalkboard, but flows

very nicely using the computer and the handouts of the presentation that each student will receive.

2) Enable the student to see the relationship between the physical problem and the mathematical graph. Many precalculus students do not see the connection between an algebraic function and its graph, nor are they able to interpret the graph in the context of the problem. By showing the box change shape as the side, "x", of the square cut from each corner increases, the student can really see that the volume depends on "x". Then by having the box displayed simultaneously with the graph of the volume function, and having the graph change as the box changes, the student has a concrete picture of the relationship between the volume of the box and the graph of the volume function. This muti-media animation forms the core of this presentation.

3) Emphasize the importance of having a strategy when approaching application problems, and then following that plan through to its finish. The plan I utilized in this problem will apply to a wide variety of applications. I carefully stated the strategy, saw it through to its conclusion, and then summarized it again at the end.

PRESENTATION 3

TRIG FUNCTIONS AND THE UNIT CIRCLE

GRAPHS OF THE SINE AND COSINE

This presentation is suitable for a trigonometry, precalculus, or first semester calculus class.

The presentation is intended to accomplish the following:

1) Provide a visual approach to the definition of the sine and cosine of real numbers, as opposed to the trig functions of angles. Using only the chalkboard, it has always been very difficult for me to describe a number line wrapping around a unit circle in a manner that students could comprehend. The animations that I was able to develop in MAPLE and PowerPoint provide a clear picture of the process of "wrapping" and its relationship to the definitions of the sine and the cosine of real numbers.

2) Develop how the graphs of the trig functions are related to their definitions in terms of the unit circle. Students at the trigonometry, and even the precalculus, level have had little experience with mathematical

graphs. They view the graph as a self-contained entity, with no relation to anything else, and they basically either memorize the shape or mindlessly plot points. To be able to have the unit circle traveling around the number line while the graphs of the sine and cosine are being traced out simultaneously on the same screen will provide a concrete connection between the points on a unit circle and the graphs of these trig functions.

3) Provide a set of notes on the unit circle with the diagrams drawn to scale, which neither I nor the student could do "by hand". Also, the diagrams and animations are drawn very quickly and accurately with the computer, whereas even rough sketches at the chalkboard would take an inappropriate amount of class time.

PRESENTATION 4

SEVERAL VARIABLES

A VISUAL OVERVIEW

This presentation is designed for use in a third semester calculus course.

The presentation was designed to accomplish the following:

1) Illustrate the similarities between what the student has learned already in the first two semesters of calculus and what that student will be learning in the third semester class. Thinking in three dimensions is difficult for many students, and they are easily intimidated by the notion of studying functions of several variables. Relating the topics in this course to topics they have already mastered in the first two courses alleviates their anxieties and creates a positive environment in which to proceed. It also provides a nice perspective of how certain areas of mathematics all fit together.

2) Provide a "wow" experience for the student. The functions they have studied in the past are visually bland when compared with the

pretty surfaces they will encounter in the several variable course. This grabs the student interest from day one of the class.

3) Provide a means by which students can really visualize the surfaces they will be studying. The pictures I draw at the chalkboard pale in comparison to the pictures that MAPLE can draw. Also, even if I mange to produce a reasonable image, the students are often unable, especially in the beginning of the class, to reproduce that image in their notes. This presentation will let them see the pictures, and the handouts generated will be a valuable additon to the students' notes.

4) Provide a key picture of many of the major topics that we will explore in the third semester course. I have developed a MAPLE worksheet with interactive images related to some of the topics. The student will be able to use this worksheet throughout the semester on the student computers to assist in the visualization of the idea we are studying at the moment.

CONCLUSIONS

During my sabbatical I have been able to accomplish everything in my initial proposal. First, my knowledge of computer software, in particular MAPLE, EXCEL, and PowerPoint has been vastly increased, and I now feel very comfortable with these programs. I have no doubt that I will continue to use software to develop innovative pedagogical techniques to use with our students. The time I was allowed, under this sabbatical, was crucial to my learning. Without the sabbatical, I would not have had the time to really explore the technology.

Second, I have developed four presentations that I and my colleagues can use in the classroom, thus affording our students a way to visualize concepts in a manner not previously available to those students. Using multi-media, a mathematical problem can be studied and solved very efficiently by several different types of strategies (numerical, algebraic, graphical, and verbal). This exposes the student to a variety of problem-solving techniques, all of which are crucial to developing critical thinking. In addition, these techniques are united in the presentations, thus giving an overall picture, as opposed to seemingly disjoint pieces. Students often see the algebra as separate from the graph, and do not relate the two. By intertwining the strategies employed, the presentations demonstrate that the algebraic and the graphical are just two ways to view the same problem.

Third, in deciding which presentations to pursue, I examined the entire curriculum for the precalculus/calculus sequence. I tried to decide what concepts were really important for enabling students to achieve success in these areas. In the first three presentations, which are aimed primarily at the precalculus level, I concentrated on one of these concepts in particular; namely, relating mathematical graphs to algebraic and numeric problem-solving, and interpreting these graphs. In the fourth presentation, directed at the third semester calculus student, I had two main goals. One was to unify the seemingly diverse topics the student has encountered in calculus so far, and the other was to enable the student to visualize three dimensional graphs through the use of the computer algebra system MAPLE.

The value of my sabbatical to the college is many-fold. On the simplest level, I have provided the college with four classroom presentations that have been developed with student success and learning at the forefront. These presentations offer alternative and innovative methods of instruction, previously unavailable to my students.

In addition, the college now has an additional instructor in the mathematics area who feels comfortable with using software to develop future projects for the classroom. I can be a resource to my colleagues, who may be attempting to incorporate multi-media into their classes.

Finally, by participating in this sabbatical, I have demonstrated a commitment to education as life-long learning. Before the sabbatical, I was fairly ignorant in the use of MAPLE, PowerPoint, and EXCEL. I had to teach myself, using manuals, resource texts, and on-line help. This has and will continue to be an example for my students, some of whom were very enthusiastic that I was undertaking this venture. I now view my classes with a renewed enthusiasm for not just teaching the student how to master topics outlined in a course syllabus, but also for empowering that student with the ability to learn in general.

APPENDICES

APPENDIX I

EXPONENTIAL FUNCTIONS

Financial Applications

Classroom Presentations
Mary Chabot
Professor, Mathematics
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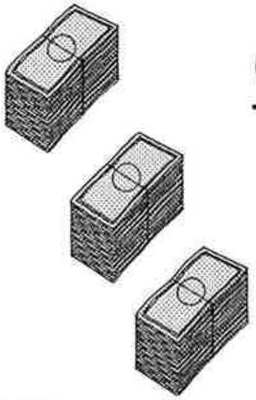
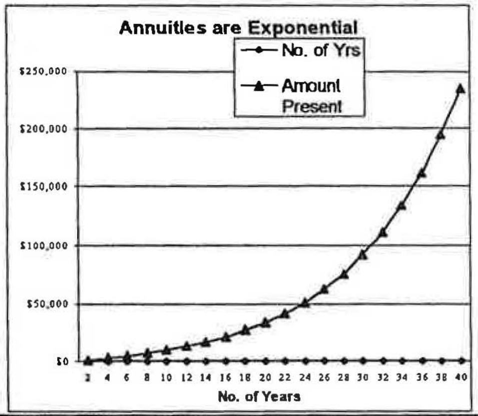
Exponential Functions Financial Applications





Compound Interest

Annuities

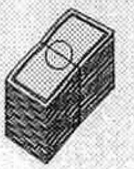
Amortized Loans

Classroom Presentations
Mary Chabot
Professor, Mathematics
Mt. San Antonio College

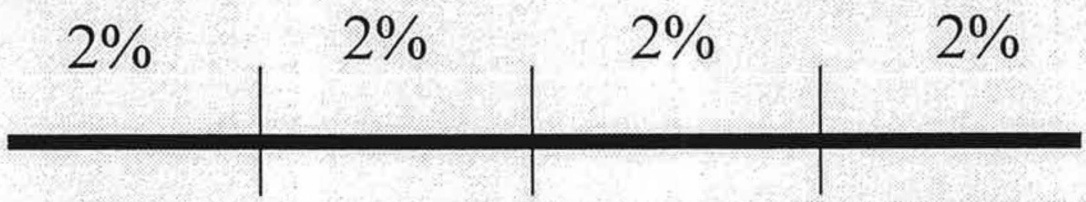
Geometric Sequences and Compound Interest

Compound Interest

- Invest $2500 at annual interest rate 8% compounded quarterly



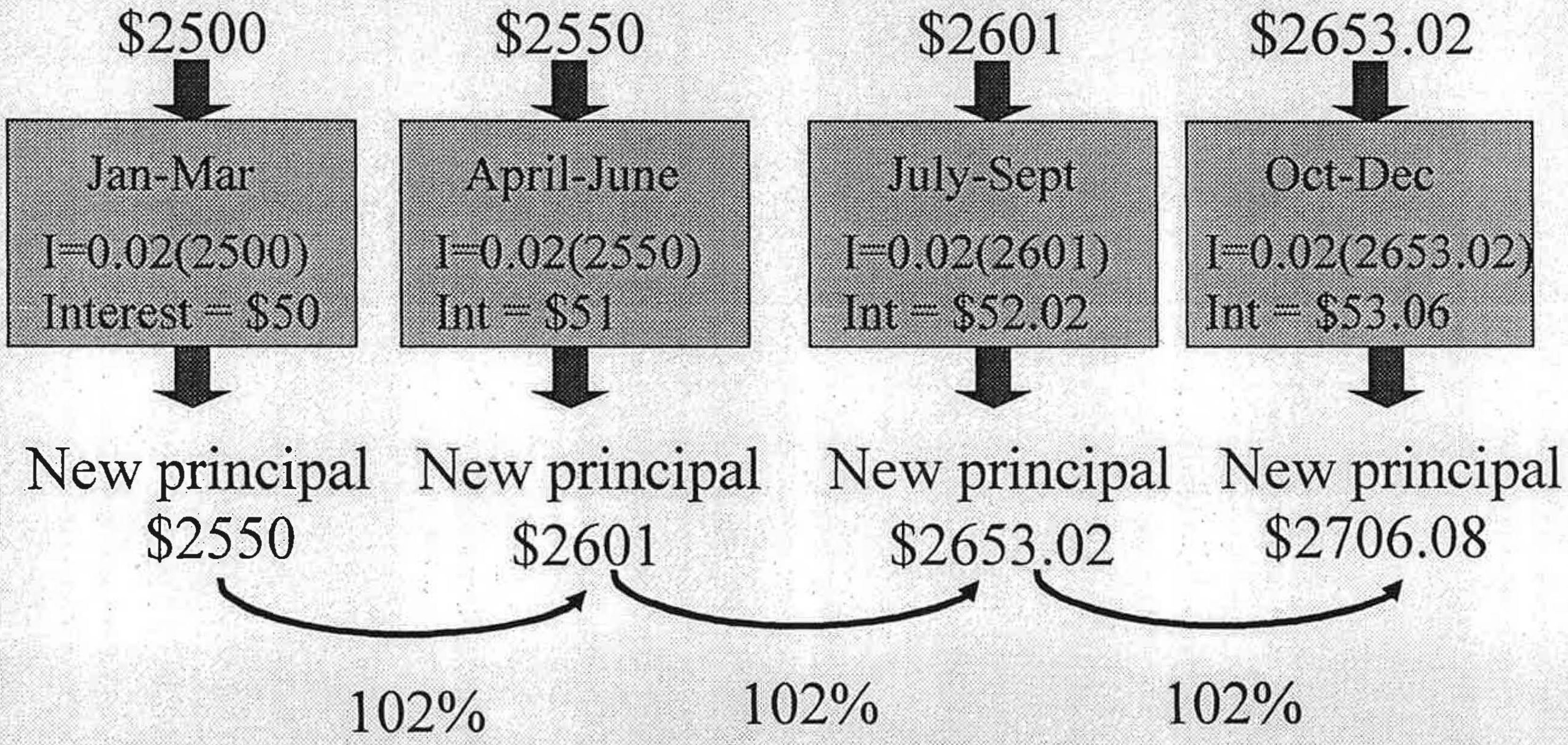
One year



- The interest rate for each quarter is 2%

Compound Interest

- Start with $2500 on January 1.



$2500	$2550	$2601	$2653.02
Jan-Mar	April-June	July-Sept	Oct-Dec
I=0.02(2500) Interest = $50	I=0.02(2550) Int = $51	I=0.02(2601) Int = $52.02	I=0.02(2653.02) Int = $53.06
New principal $2550	New principal $2601	New principal $2653.02	New principal $2706.08

102% 102% 102%

Compound Interest

- The amount **A** in the account at the end of each period forms a geometric sequence:

$$\$2500, \xrightarrow{1.02} \$2500(1.02), \xrightarrow{1.02} \$2500(1.02)^2, \xrightarrow{1.02} \$2500(1.02)^3, \xrightarrow{1.02} \$2500(1.02)^4, \ldots$$

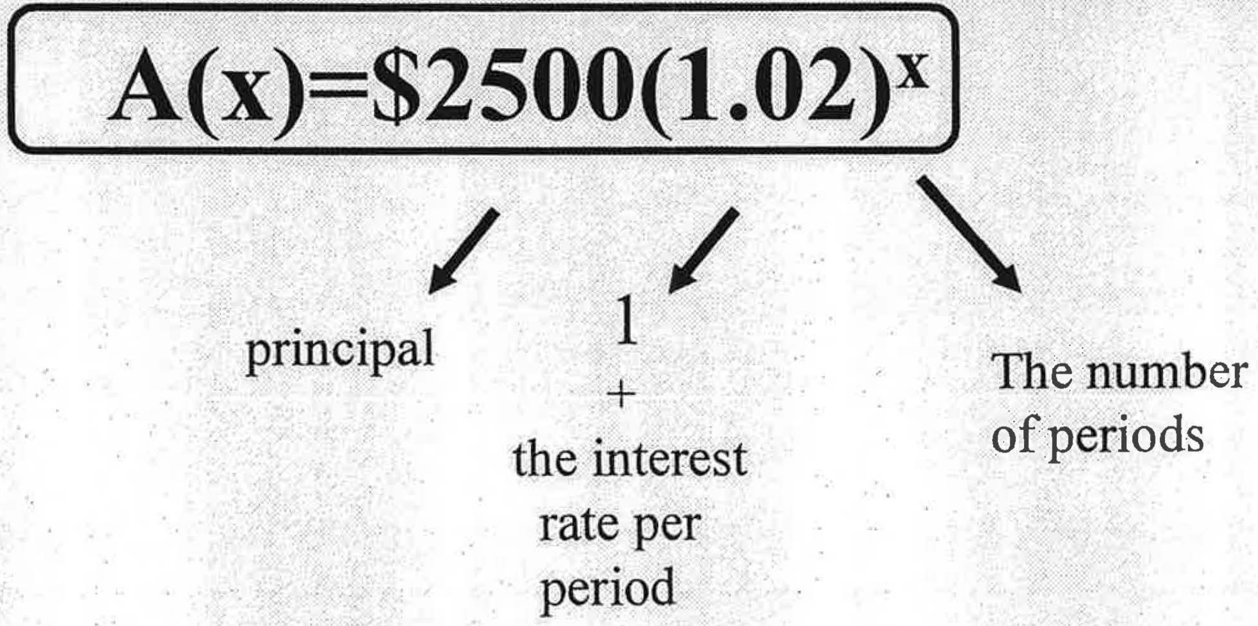
A(0) A(1) A(2) A(3) A(4)

$$\boxed{A(x)=\$2500(1.02)^x}$$

Exponential function, base 1.02

Compound Interest

- Generalizing:



$$\boxed{A(x)=\$2500(1.02)^{x}}$$

principal

1
+
the interest
rate per
period

The number
of periods

Note: If compounded quarterly for 5 years, the number of periods is (4)(5)

If compounded n times a year for t years, the number of periods is **nt**

Compound Interest

If $\mathbf{P}$ dollars is invested at annual rate $\mathbf{r}$, compounded $\mathbf{n}$ times a year, the amount $\mathbf{A}$ in the account after $\mathbf{t}$ years is :

$$\boxed{A(t) = P(1+r/n)^{nt}}$$

This is still an exponential function with base >1 and non-negative exponents; it will continue to grow more steeply as time increases .

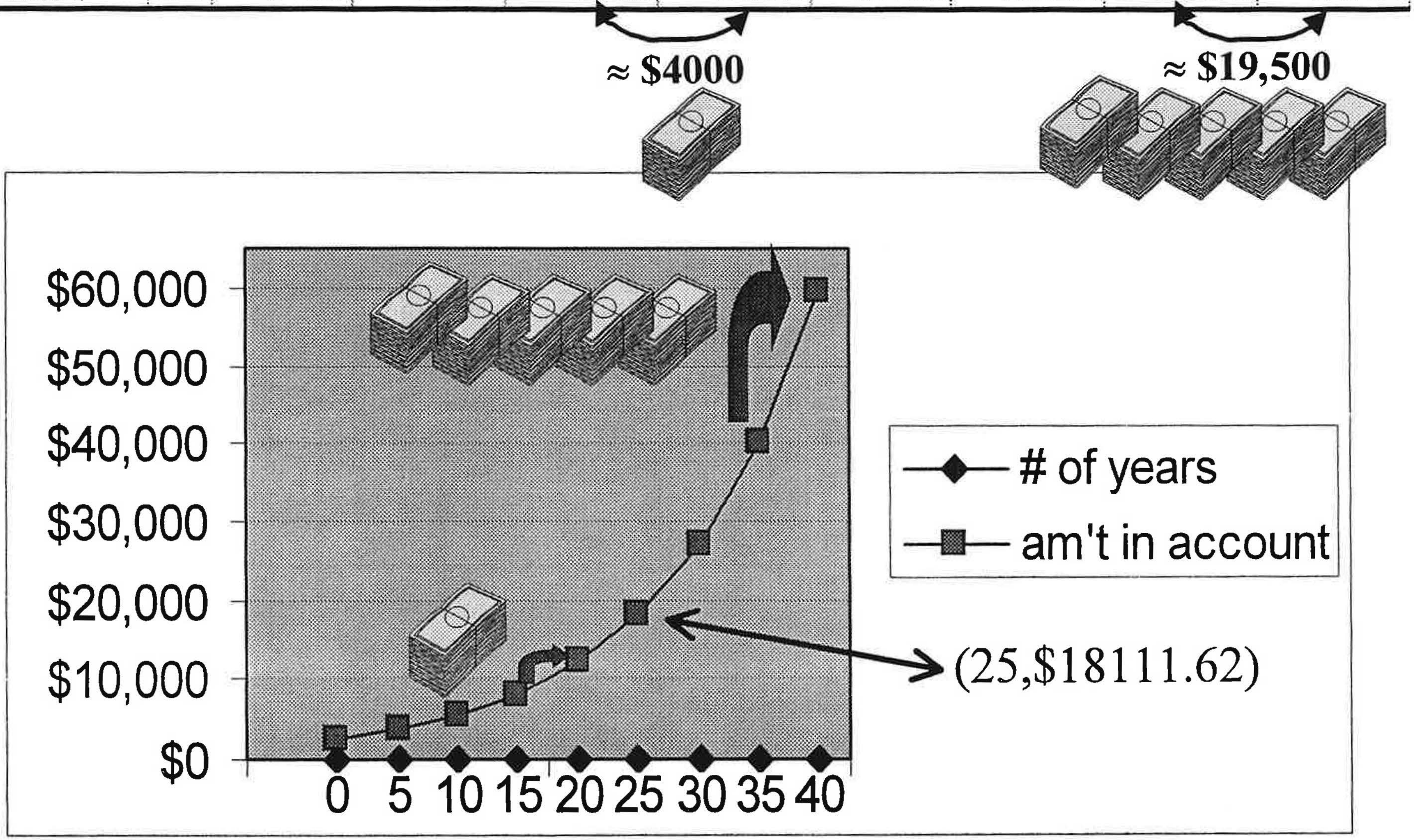
Invest \$2500 at annual rate 8% compounded quarterly for 40 years.

$$A(40) = 2500(1+.08/4)^{4(40)}$$

$$A(40) = \$59{,}424.77 \quad !!!!$$

$2500 at annual rate 8% compounded quarterly

# of years	0	5	10	15	20	25	30	35	40
am't in account	2500	3714.87	5520.10	8202.58	12188.60	18111.62	26912.91	39991.16	59424.77

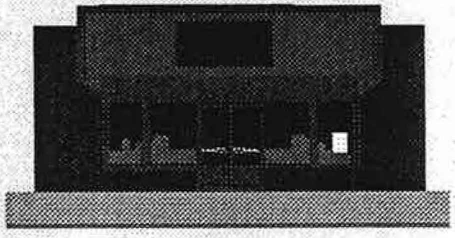


Classroom Presentations
Mary Chabot
Professor, Mathematics
Mt. San Antonio College

Geometric Series and Annuities

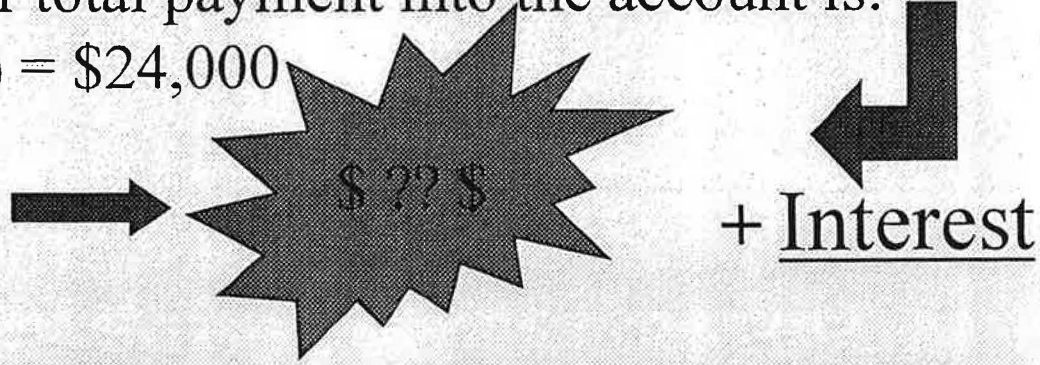
Annuities

- Deposit $50 per month into an account with annual interest rate 9%, compounded monthly



After 40 years, your total payment into the account is:

$50(12)(40) = $24,000



$24000

+ Interest

How much money is in the account ?

Annuities

Assume the payment is due at the end of the period.
The last payment receives no interest.
The next to the last payment receives interest for one month, the payment before that receives interest for 2 months, etc.

Last	Next to last	Before that	Etc.
\$50	$\$50(1+.09/12)^1$	$\$50(1+.09/12)^2$	

The amount present in the account after the 480 payments is:

$$\$50 + \$50(1+.09/12)^1 + \$50(1+.09/12)^2 + \$50(1+.09/12)^3 + \ldots + \$50(1+.09/12)^{479}$$

This is the sum of a part of a geometric sequence, with first term \$50 and common ratio (1+.09/12).

Annuities

Formula: $S_n = \frac{a_1(1-r^n)}{1-r}$ Sum of the first **n** terms in a geometric sequence

$a_1 = \$50$; $r = (1+.09/12)$; $n = 480$

Future Value = $\frac{\$50(1-(1+.09/12)^{480})}{1-(1+.09/12)}$

= - .09/12

Future Value = $\frac{\$50((1+.09/12)^{480}-1)}{.09/12}$ = \$234,066.01

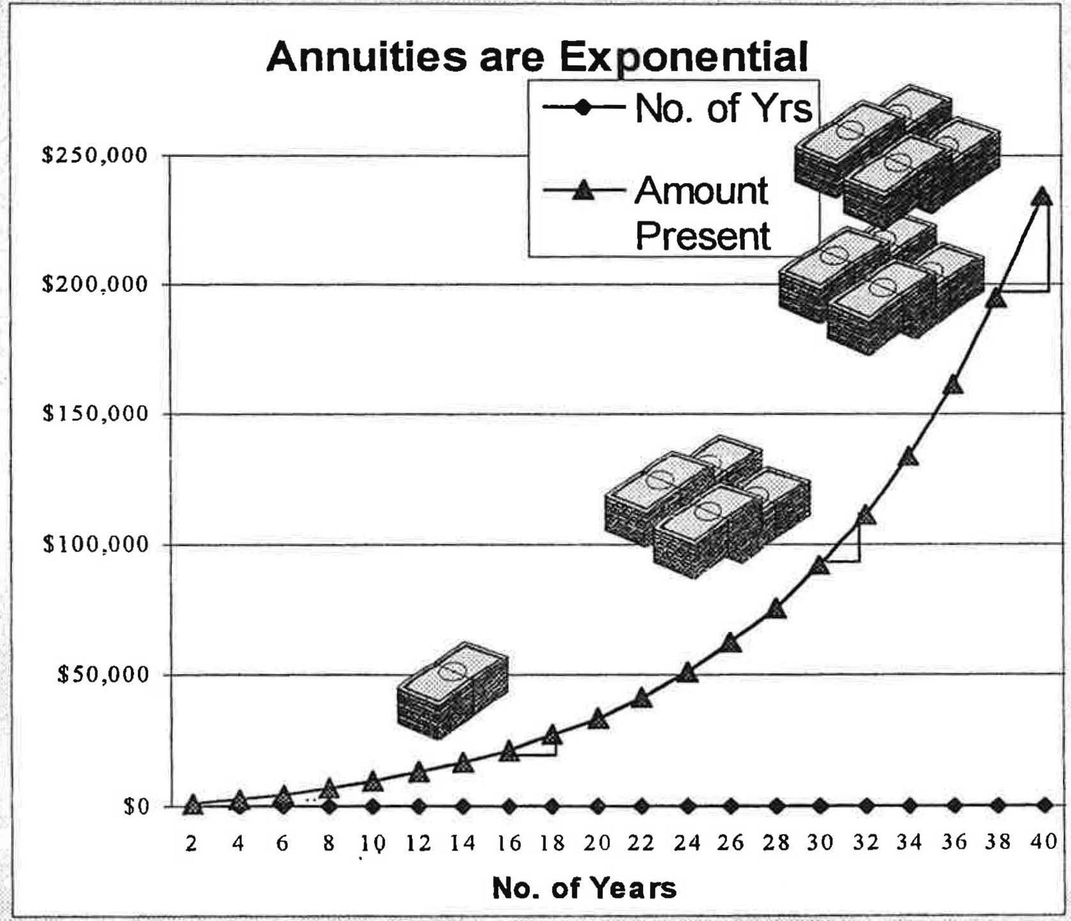
Future Value = $\frac{\text{payment}((1+\text{rate per period})^{\text{no. of periods}} - 1)}{\text{rate per period}}$

A-13

Annuities

Invest $50 per month at annual rate 9% , compounded monthly

No. of Yrs	*Amount Present*
2	$1,309.42
4	$2,876.04
6	$4,750.35
8	$6,992.81
10	$9,675.71
12	$12,885.58
14	$16,725.90
16	$21,320.52
18	$26,817.58
20	$33,394.34
22	$41,262.87
24	$50,676.88
26	$61,939.92
28	$75,415.19
30	$91,537.17
32	$110,825.74
34	$133,902.83
36	$161,512.59
38	$194,545.27
40	$234,066.01

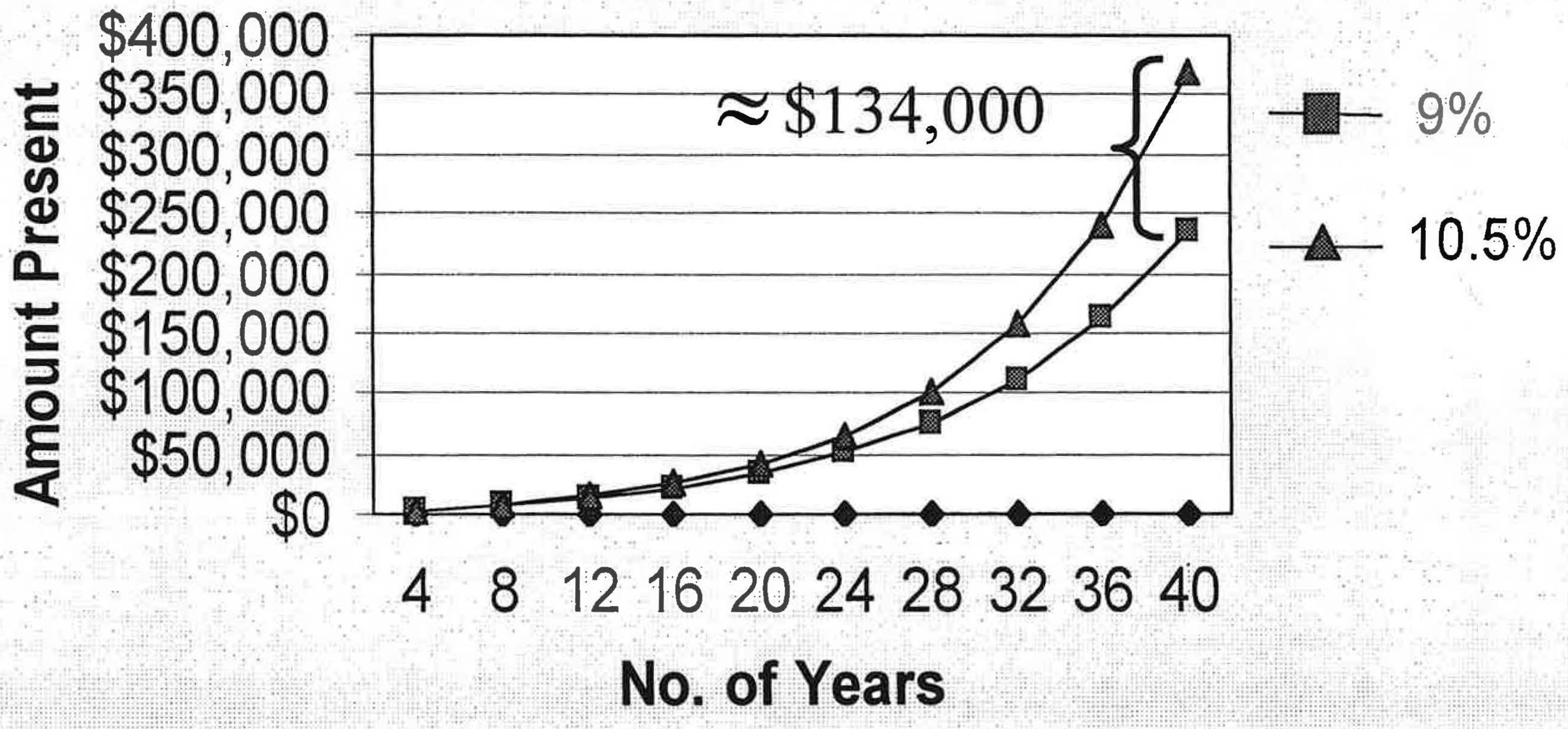


Recall: your total payment into the acc't=$24,000

Annuities

No. of Yrs	4	8	12	16	20	24	28	32	36	40
Annual rate 9%	$2,876	$6,993	$12,886	$21,321	$33,394	$50,677	$75,415	$110,826	$161,513	$234,066
Ann. Rate 10.5%	$2,967	$7,474	$14,321	$24,723	$40,525	$64,532	$101,003	$156,409	$240,580	$368,452

$50 Monthly Payment



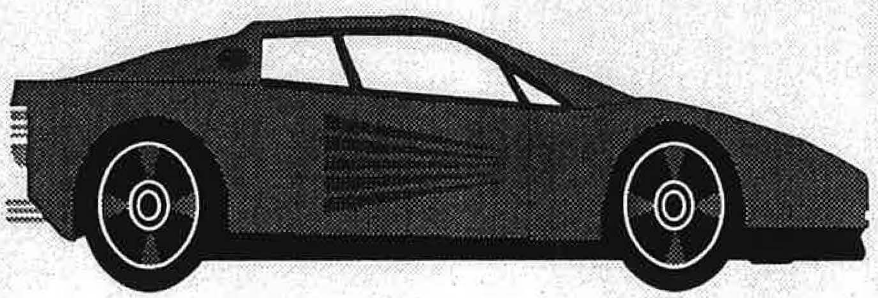
Amortized Loans

The loan amount, plus interest, is paid off in regular, equal payments

Amortized Loans

Buying a Car

You want to buy a car whose total price is $23,600.



You make a $2000 down payment, and finance the rest.

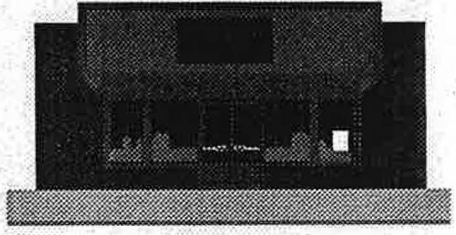
The car dealer is offering a 5 year, 6.9% loan,
to be paid off in regular monthly installments.

How much will you have to pay each month to finance the car ?

Amortized Loans

Computing the Payments

Your payment is like the payment on an annuity.



Future value of annuity = Future value of loan (include interest)

$$pymt\frac{(1+i)^n-1}{i} = P(1+i)^n$$

where P = loan amount, i = periodic interest rate, and n is the number of periods

We can solve this for "pymt".

Amortized Loans

Formula

$$payment = P\frac{i(1+i)^n}{(1+i)^n-1}$$

After the $2000 down payment, you want to finance $21600 for 5 years at APR 6.9%, paid off monthly.

P = $21600, n= 5(12) = 60, i = 0.069/12

Your monthly payment will be:

$426.69

Exponential Functions Financial Applications

THE END

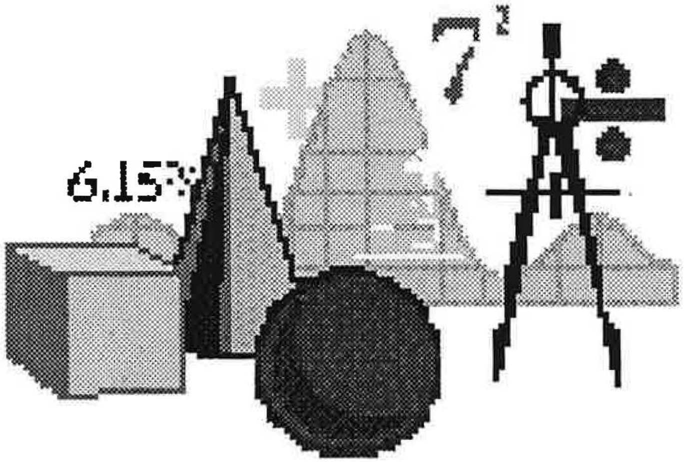
APPENDIX II

OPTIMIZATION

The Box Problem

Classroom Presentations

⏮ ◀ ▶



Mary Chabot

Professor, Mathematics

Mt. San Antonio College

OPTIMIZATION

THE BOX PROBLEM

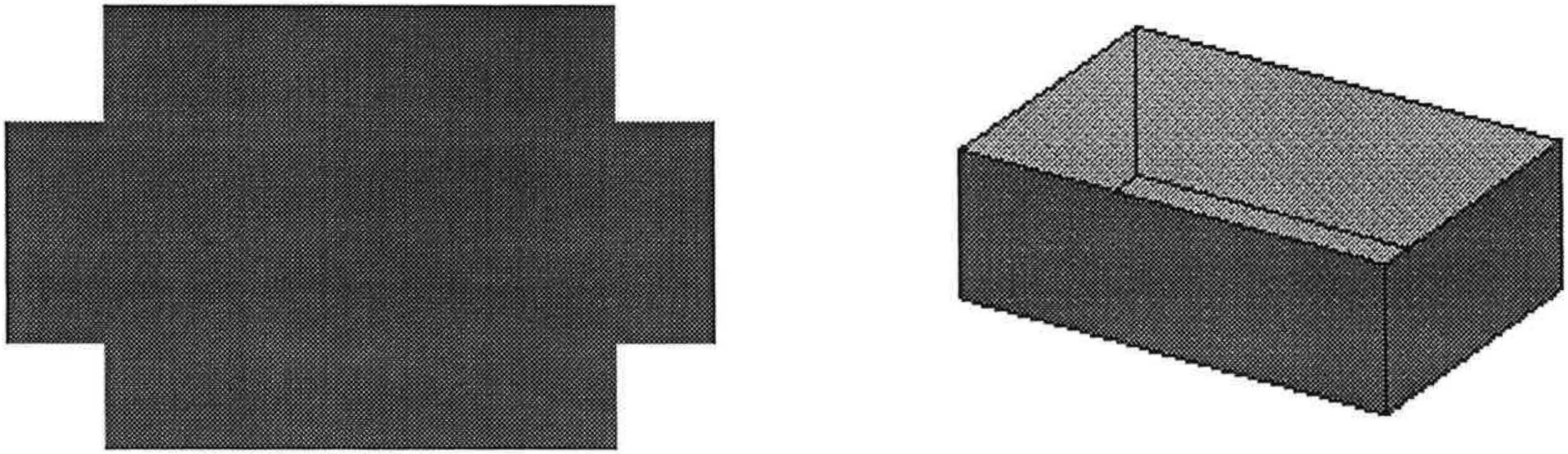


Illustration II

CONTEST !!!!

The person with the most m&m's

$$$ WINS THE MONEY $$$

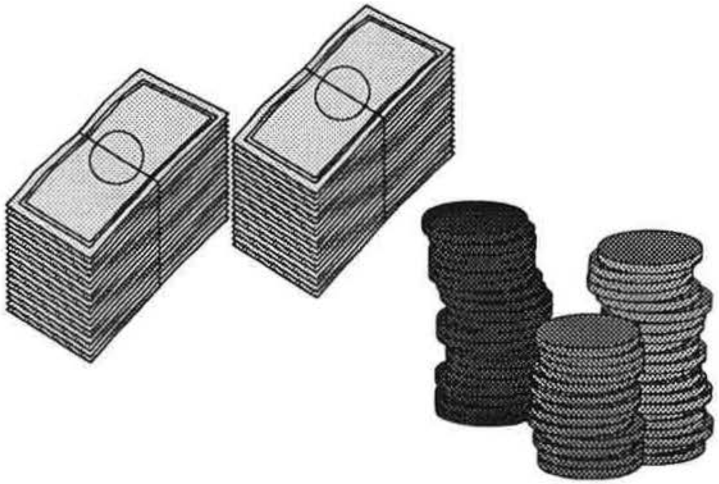
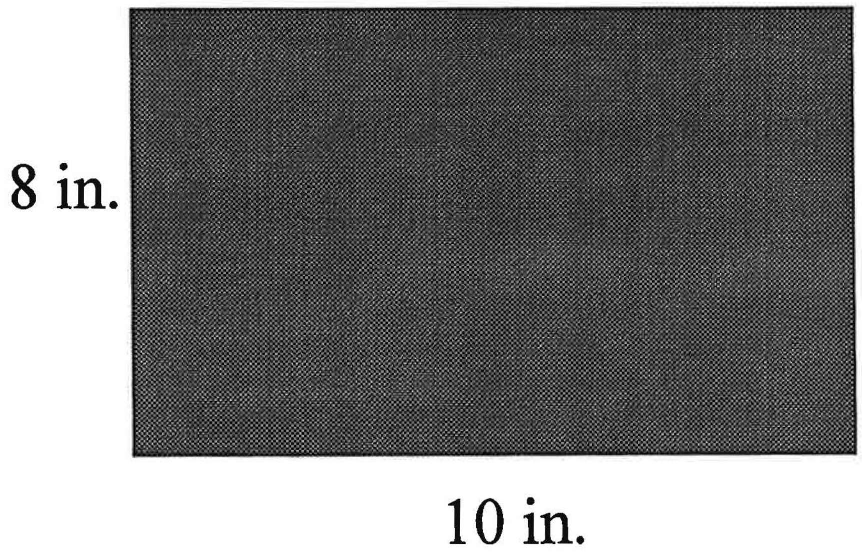


Illustration II

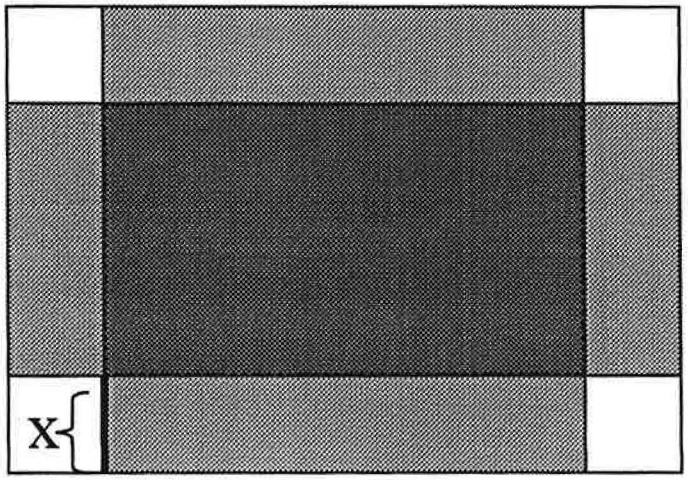
RULES

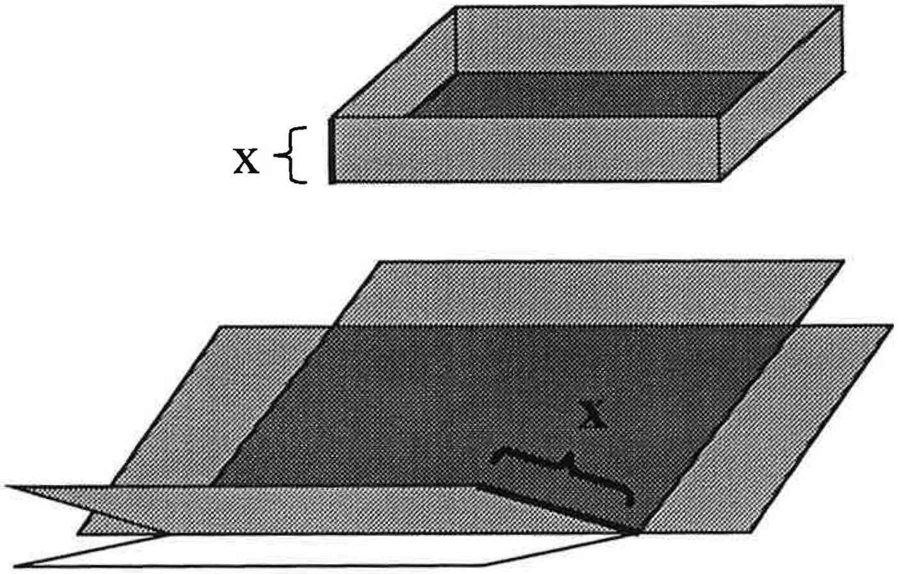
You will be given an 8 in. by 10 in. piece of cardboard



USE THE CARDBOARD TO MAKE A BOX FOR YOUR M&M'S

Cut a square from each corner, fold up the sides, to form a box.



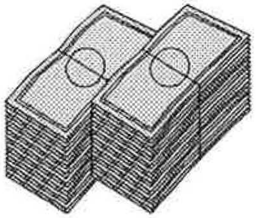


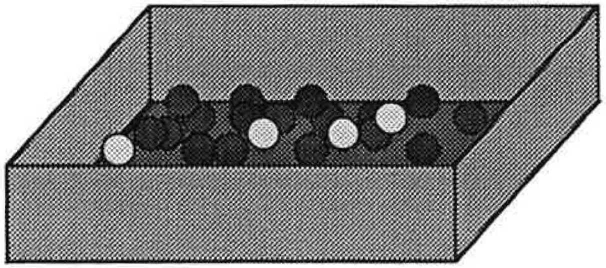
NOTE the relationship between the side of the square
and the height of the box.

Illustration II

Fill the box with m&m's

$$$ Remember, the person with the most m&m's wins the money $$$





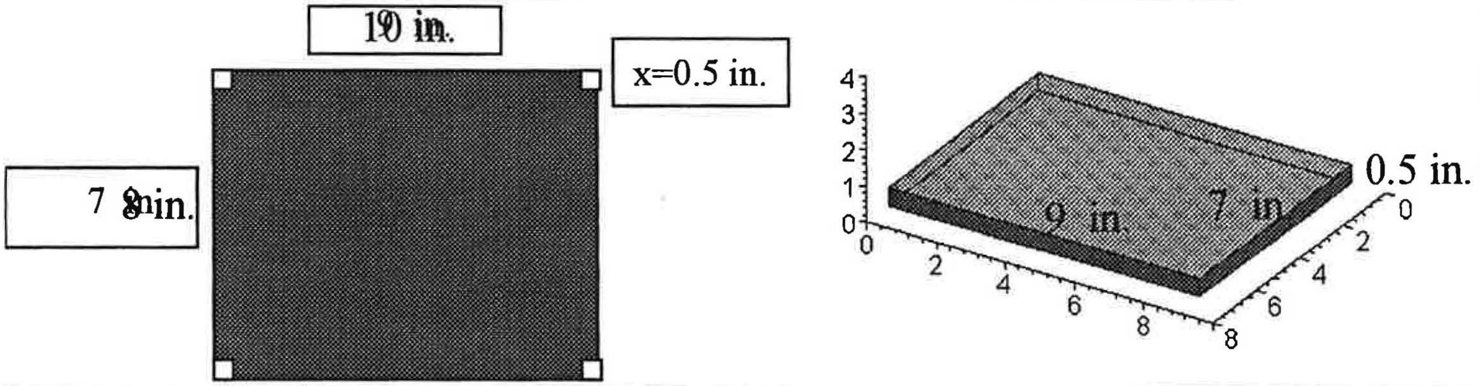
Your goal, therefore, is to maximize the volume of the box you construct.

You can try to guess what size square to cut,

OR you can use that **precalculus** you thought you'd never need **Illustration II**

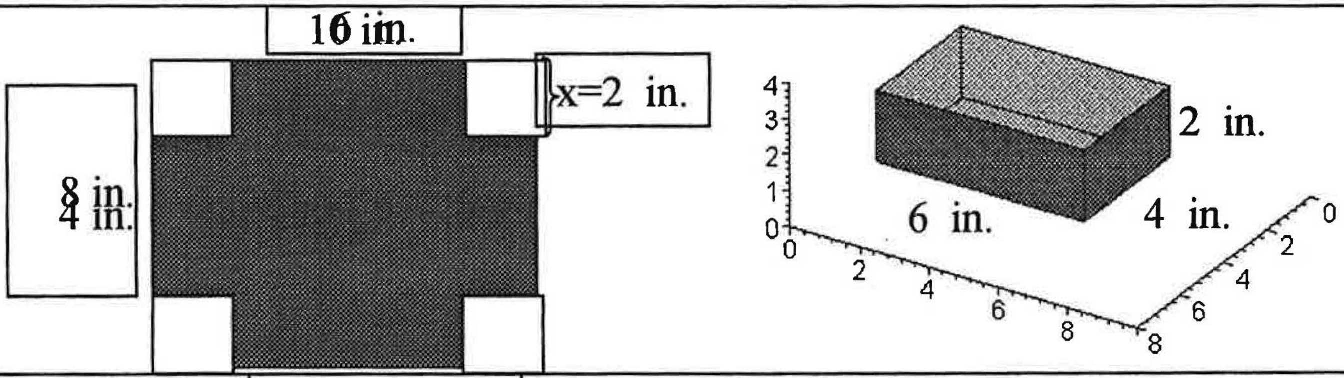
NOTE: changing the value of x changes the shape of the box

V = L·W·H



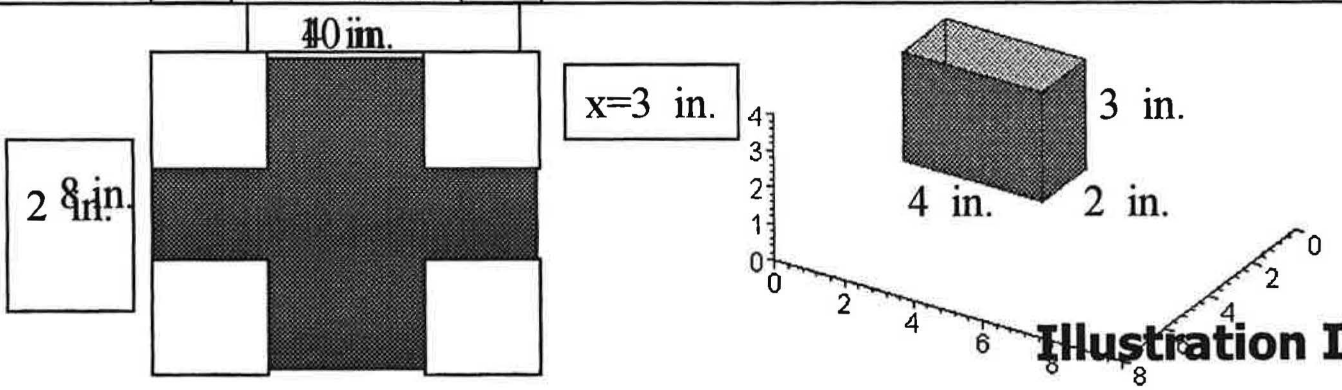
$$V = (9)(7)(.5)\ \text{in}^3$$

$$V = \boxed{31.5\ \text{in}^3}$$



$$V = (6)(4)(2)\ \text{in}^3$$

$$V = \boxed{48\ \text{in}^3}$$



$$V = (4)(2)(3)\ \text{in}^3$$

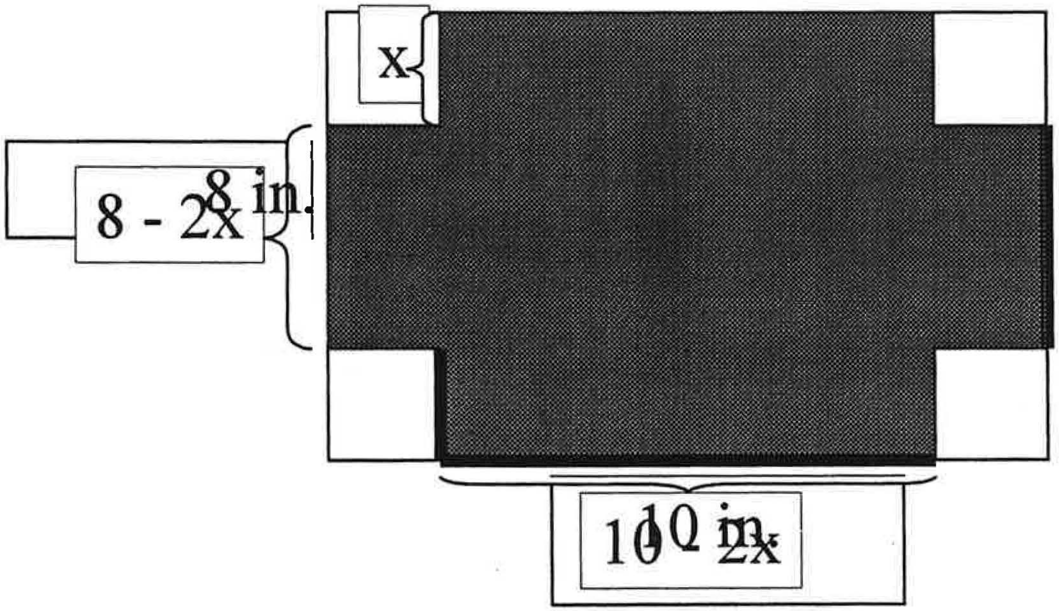
$$V = \boxed{24\ \text{in}^3}$$

Illustration II

STRATEGY

- Find a **formula** for the volume of the box as a function of x
- Find the **values of x that are acceptable** for the box
- **Graph** the volume function for the acceptable values of x
- Use the graph to approximate the value of x that will **maximize the volume**

Find the formula



V = L• W•H

$$V = (10-2x)(8-2x)(x)$$

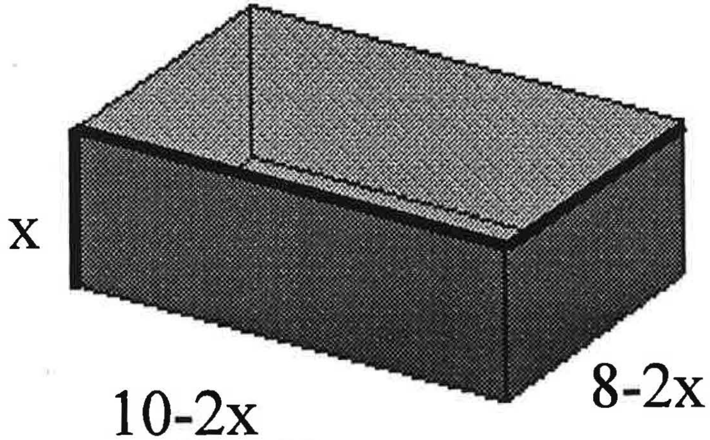
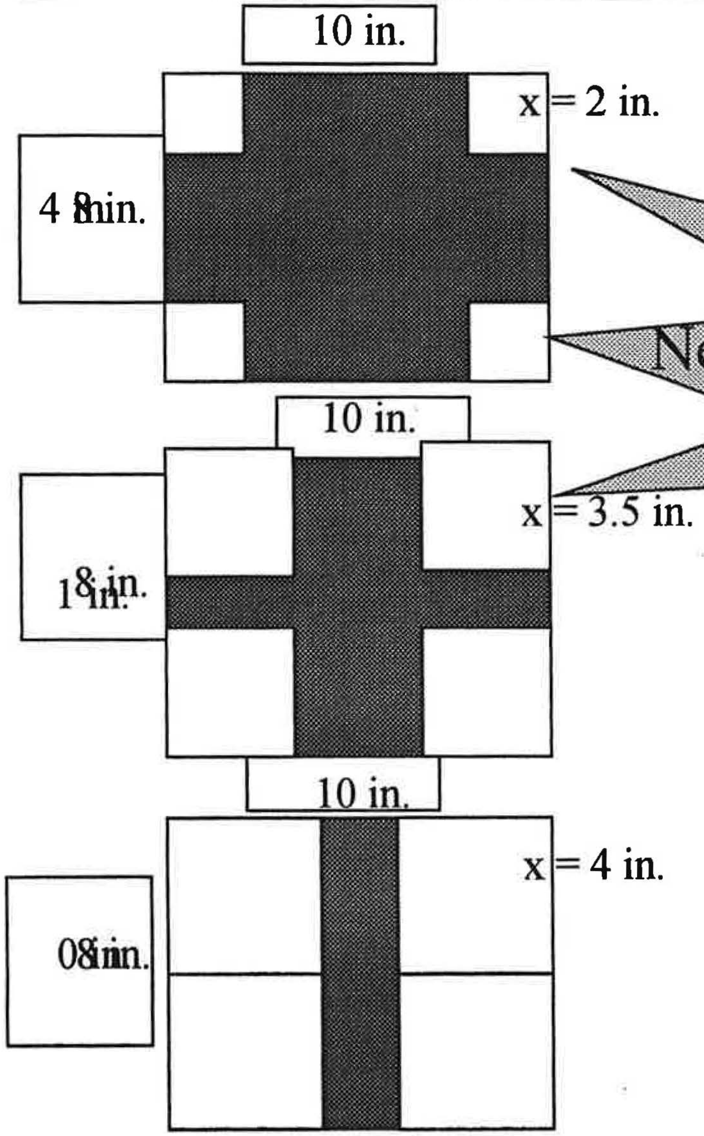


Illustration II

Acceptable values for x

$V = L\ W\ H$

Need $L \geq 0$ AND $W \geq 0$ AND $H \geq 0$



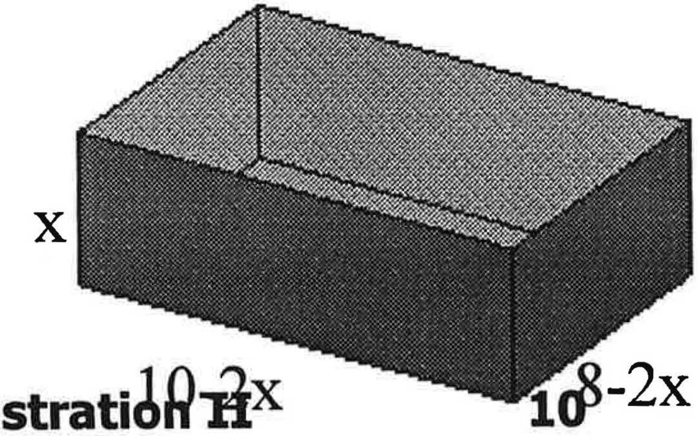
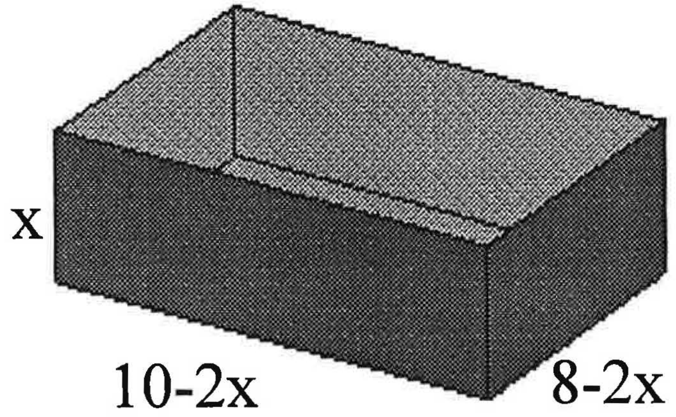


Illustration II

Solve the inequalities

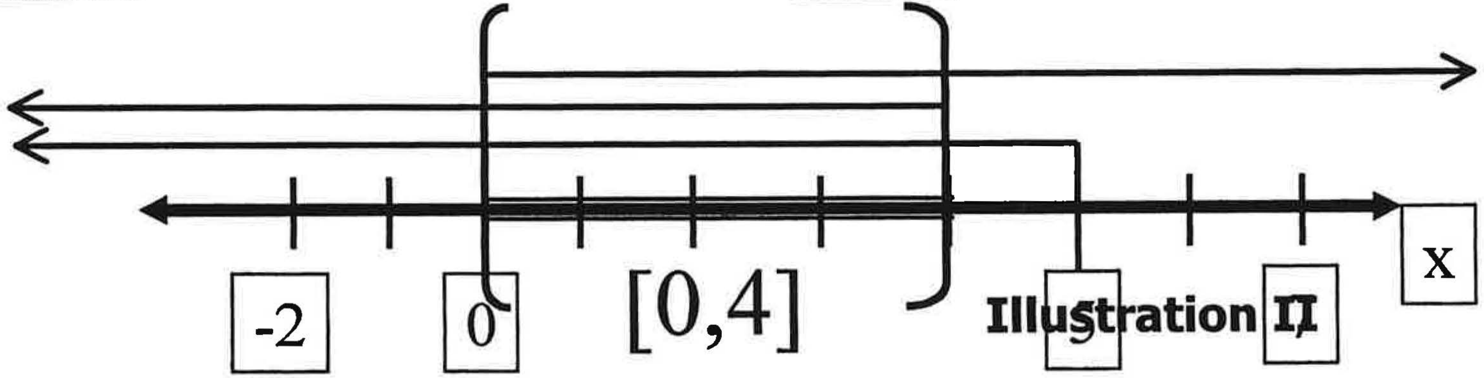
Need $L \geq 0$ AND $W \geq 0$ AND $H \geq 0$



$10-2x \geq 0$ AND $8-2x \geq 0$ AND $x \geq 0$

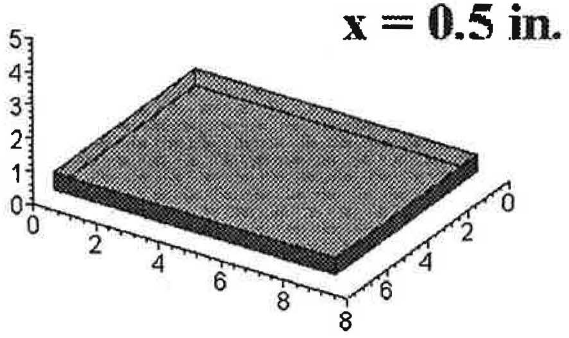
$-2x \geq -10$ $\quad$ $-2x \geq -8$

$\boxed{x \leq 5}$ AND $\boxed{x \leq 4}$ AND $\boxed{x \geq 0}$



Graph V(x) over the domain

$$V(x) = (10-2x)(8-2x)(x) \quad \text{on } [0,4]$$



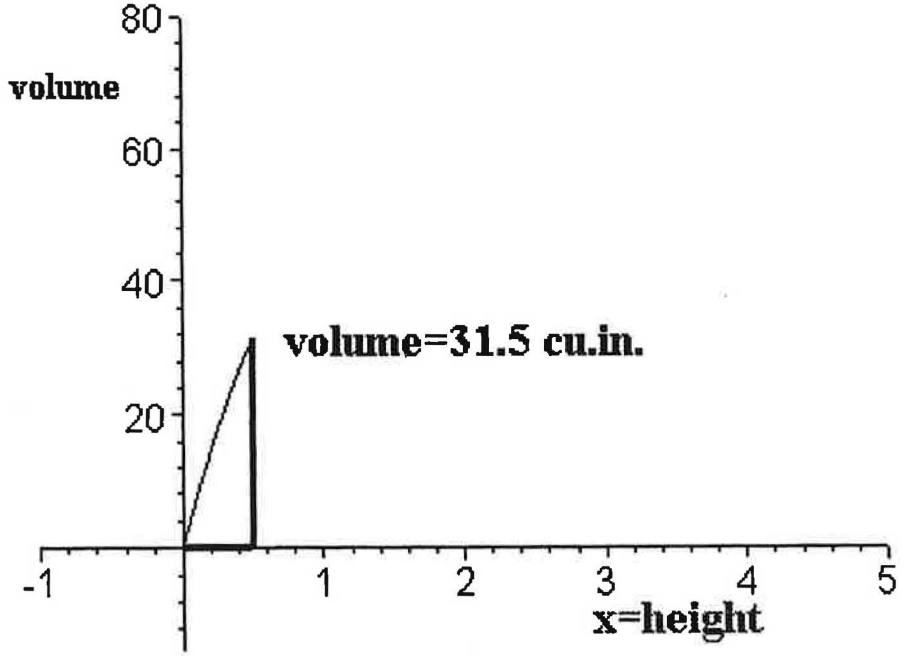
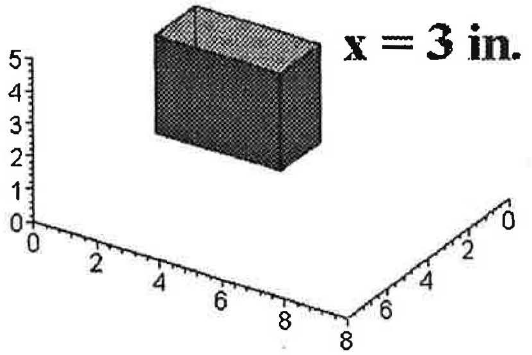
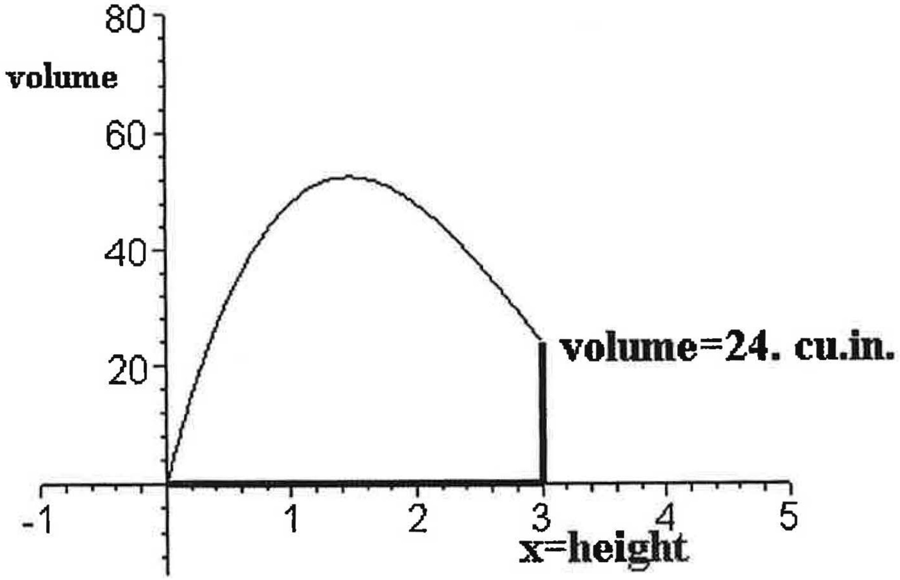


Illustration II

Graph V(x) over the domain

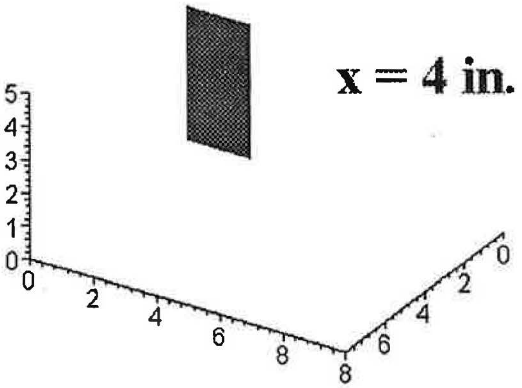
$$V(x) = (10-2x)(8-2x)(x) \quad \text{on } [0,4]$$





Graph V(x) over the domain

$V(x) = (10-2x)(8-2x)(x)$ on $[0,4]$



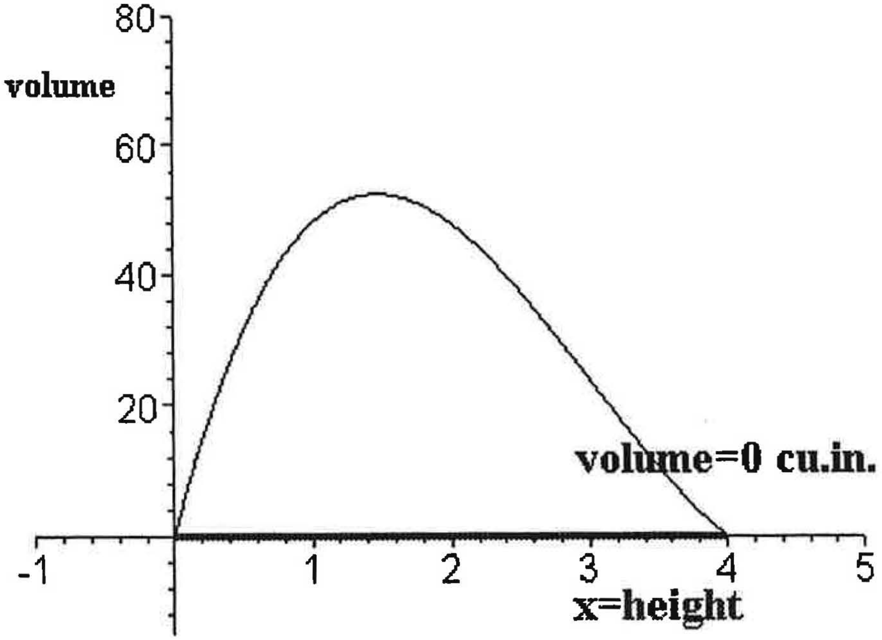
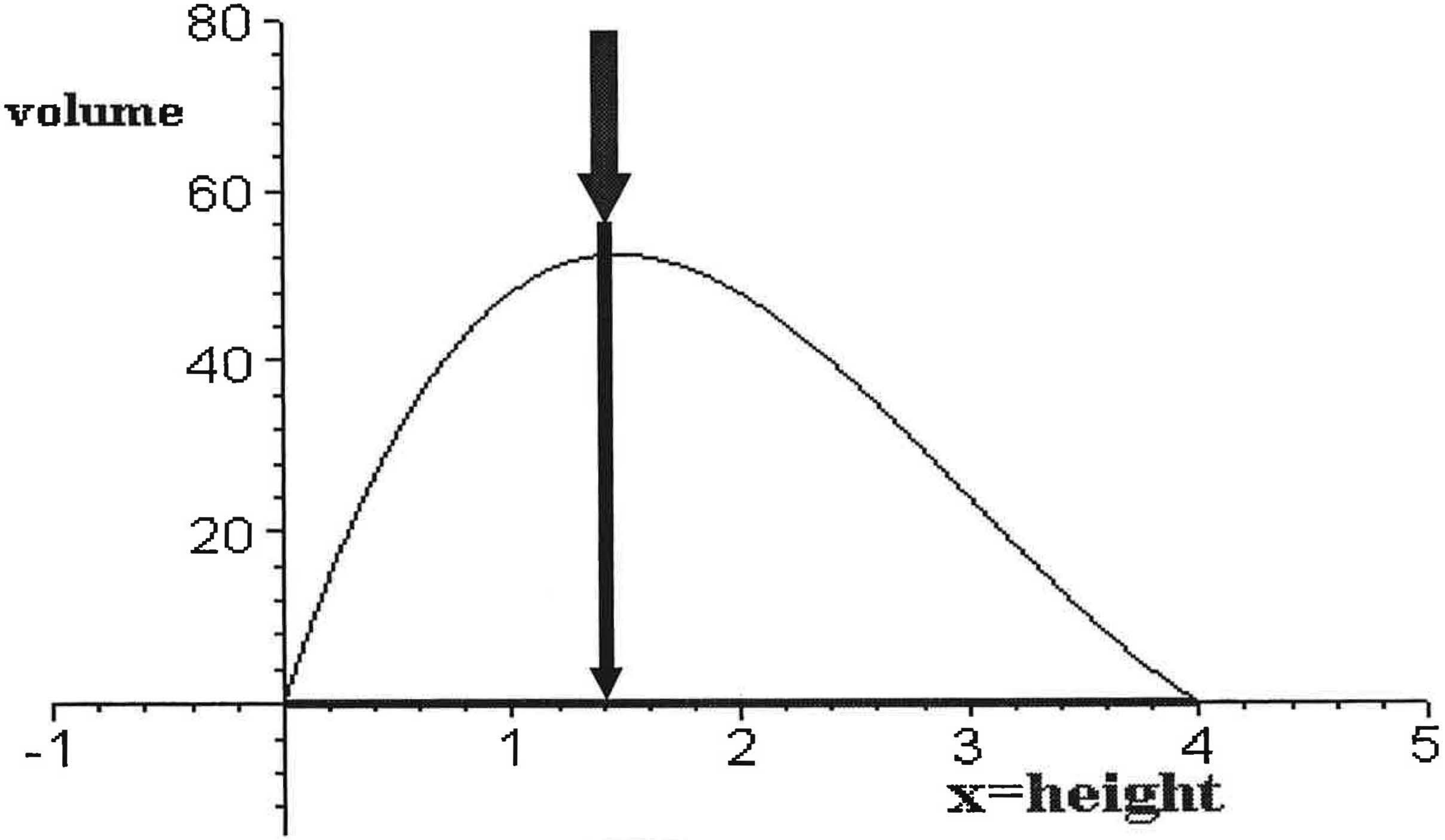


Illustration II

Maximize the Volume



x ≈ **1.4*in.***

Illustration II

Maximum Volume

$$V = (10-2x)(8-2x)(x)$$

$x \approx 1.4in.$

$$V = (10-2(1.4))(8-2(1.4))(1.4)$$

$$\boxed{V = 52.416 \text{ cu. in.}}$$

Illustration II

Do You Win the Money?

If there are 64 m&m's per cu. in. (they meant MINI m&m's), then your box will hold:

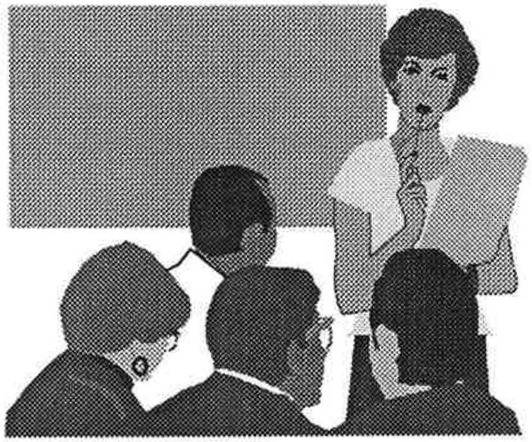
$$\frac{64 m\&m's}{cu.in}\cdot 52.416 cu.in. = 3354.624 m\&m's$$

$$\approx 3354 m\&m's$$

You have just found out that the prize is $0.10 for each m&m in the winning box (that's $335.40, if you win)

BAD NEWS AND GOOD NEWS !!!

Bad News

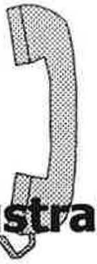


Your friend has taken that calculus course

where she learned how to find the EXACT value of x that produces the maximum volume!

She determined that the optimum value of x is $3 - \frac{\sqrt{21}}{3}$ in., which will allow her to have 3360 m&m's !!!

You have spent so much time on this problem, you decide to talk with her.



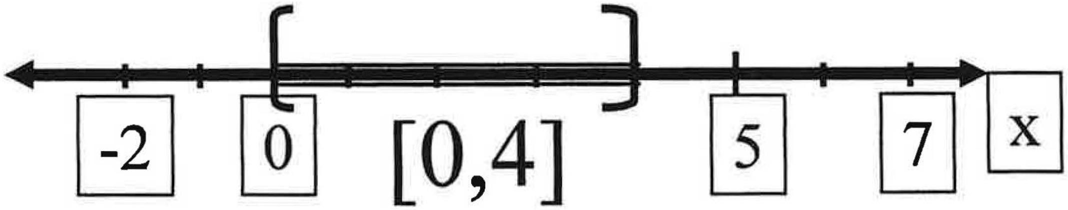
How to Win

She explains that she still had to do your work:

Find a **formula** for the volume of the box as a function of x

$$\boxed{V = (10-2x)(8-2x)(x)}$$

Find the **values of x that are acceptable** for the box



Picture a Graph of the volume function for the acceptable values of x

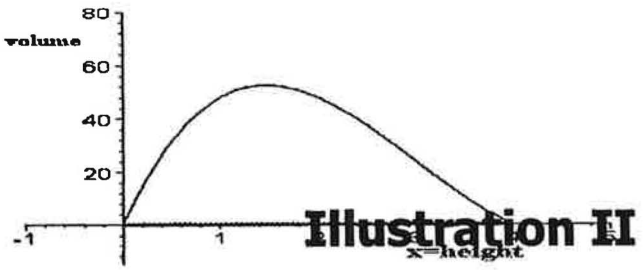
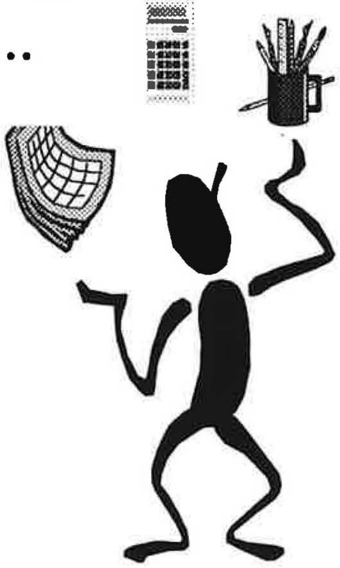


Illustration II

Calculus

Calculus provides a method for determining exact answers to optimization problems, BUT…………..



Precalculus provides the tools

that are essential for success

in calculus

Good News

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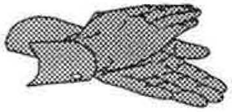
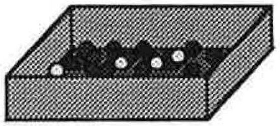
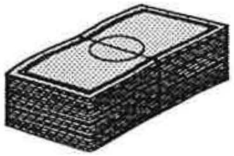
The judges were so impressed with your efforts,  they awarded you:

1) all the m&m's entered in the contest 

2) the opportunity to enroll in that calculus class 

3) a $100 consolation prize 

THE END

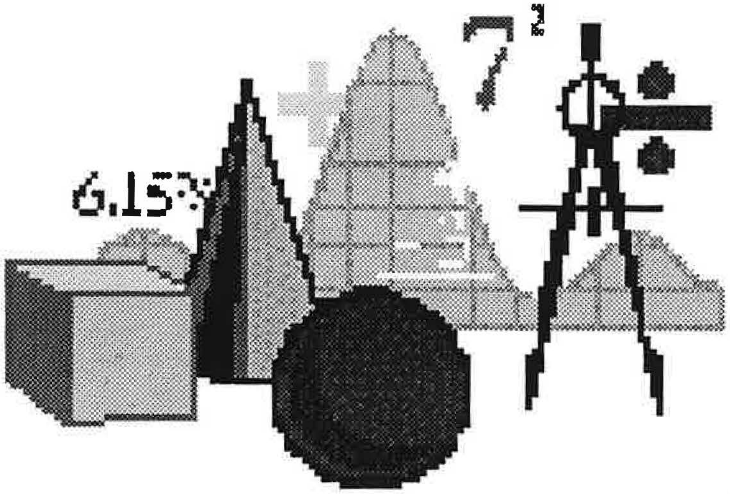
APPENDIX III

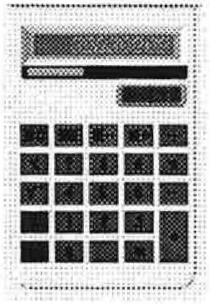
Trig Functions and the Unit Circle

Graphs of the Sine and Cosine

CLASSROOM PRESENTATIONS

Mary Chabot
Mt. San Antonio College





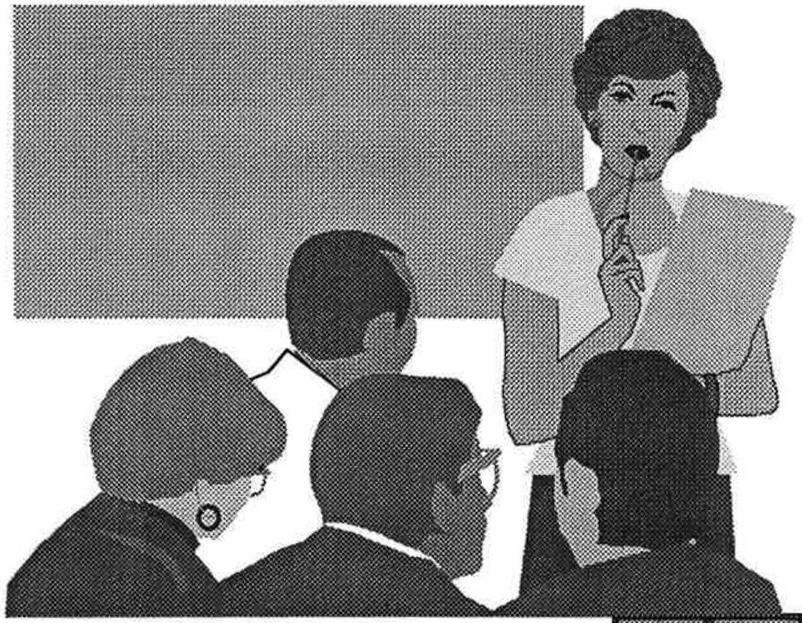
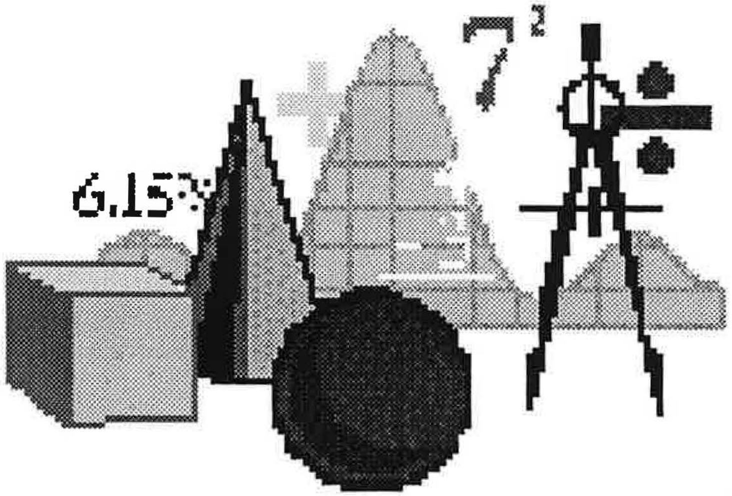
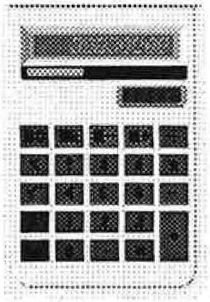


Illustration III 1

TRIGONOMETRY

- ▨ Unit Circle
- ▨ Graphs





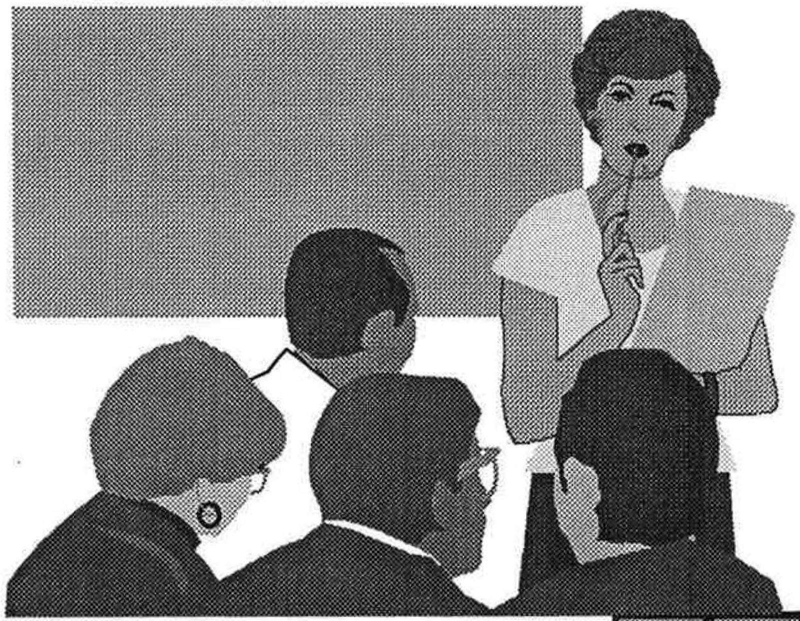


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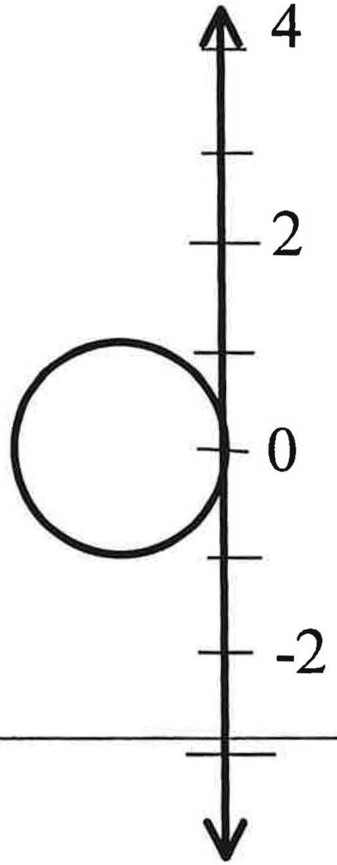
Illustration III 2

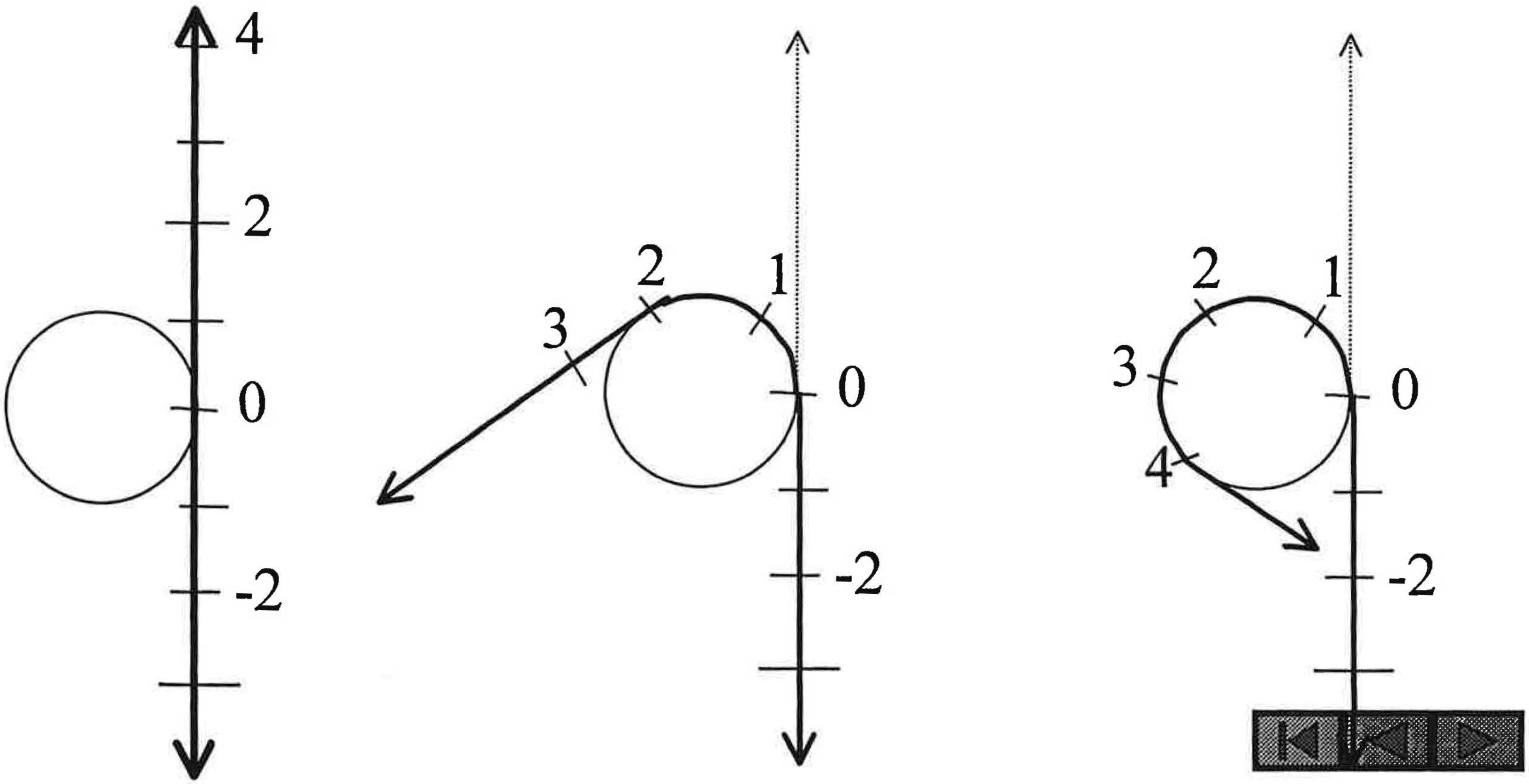
Trig functions of real numbers

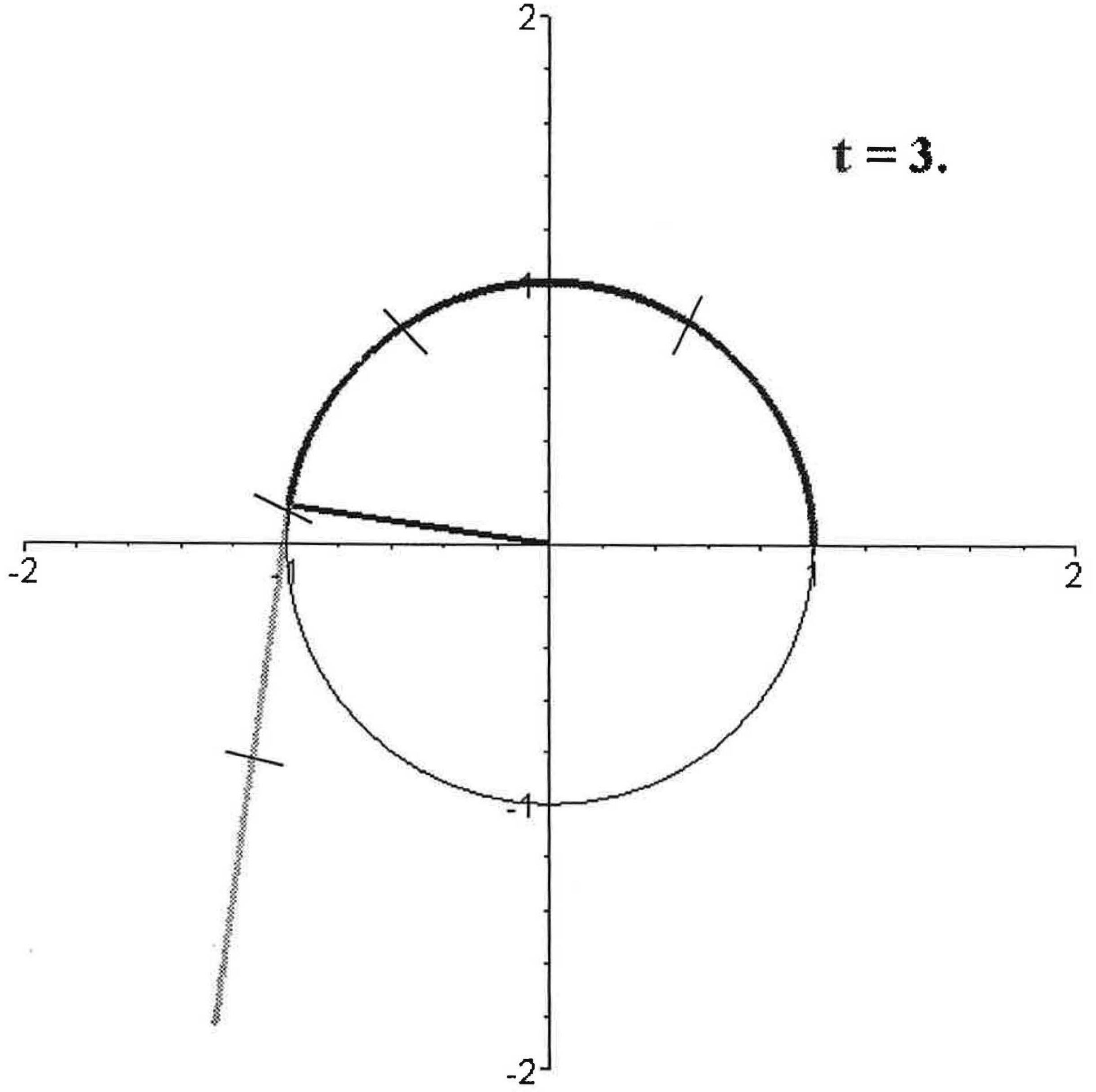
- Start with a unit circle and a number line



Wrap the number line around the unit circle

Positive real numbers wrap counterclockwise

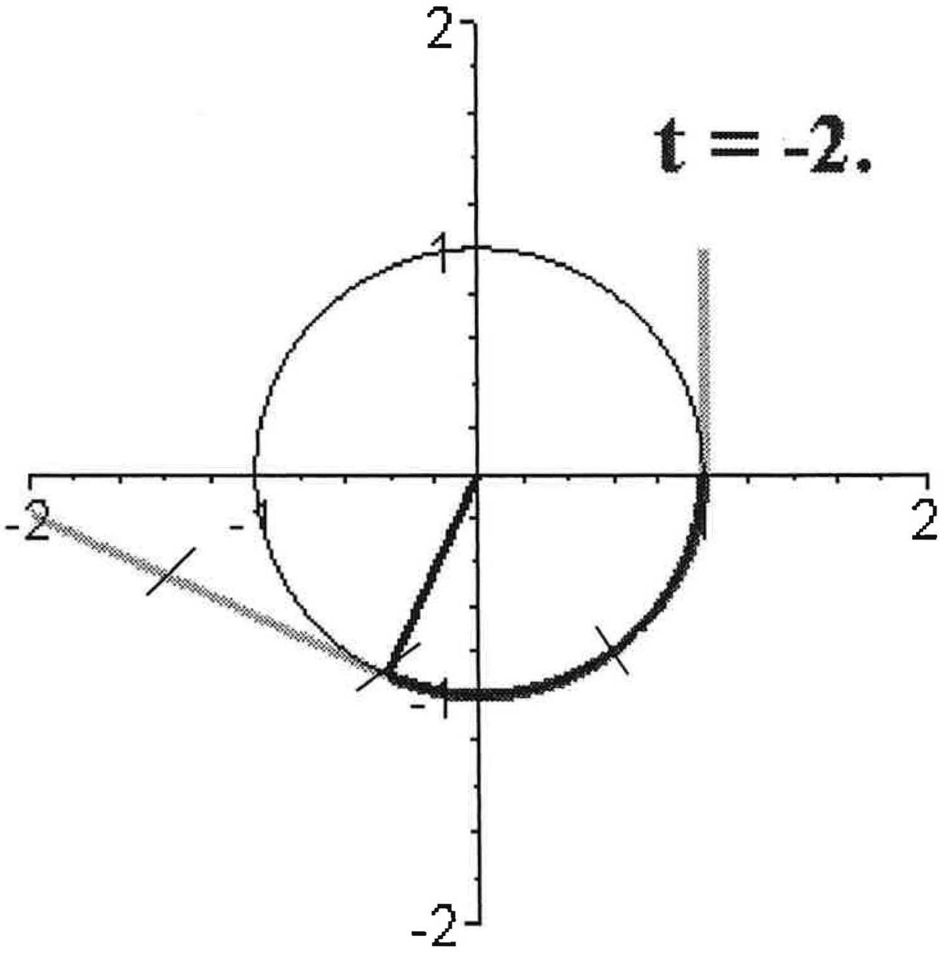




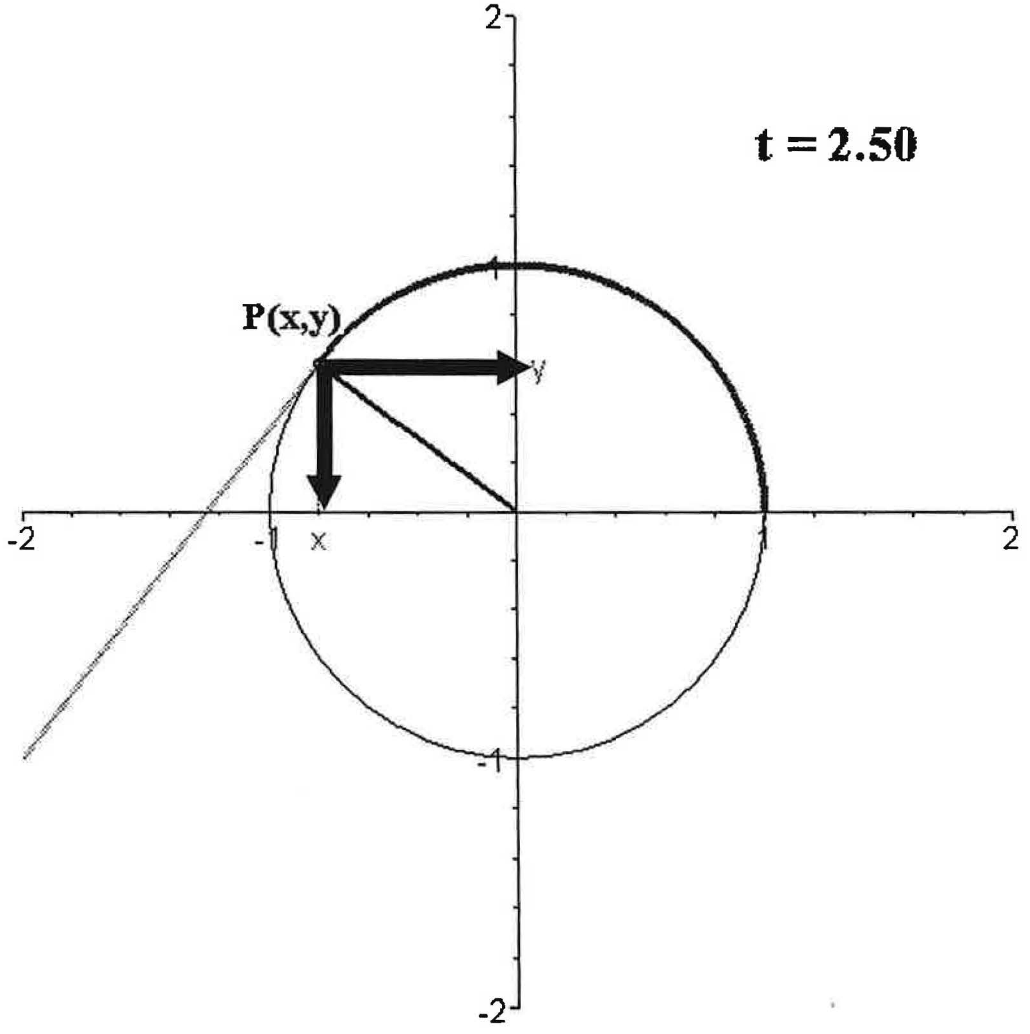
⏮ ◀ ▶

Illustration III 5

Negative real numbers wrap clockwise



Every real number "t" is thus associated with a point P(x,y) on the unit circle.



A-51

The y-coordinate of point P is the sine of “t”
sin(t) = y

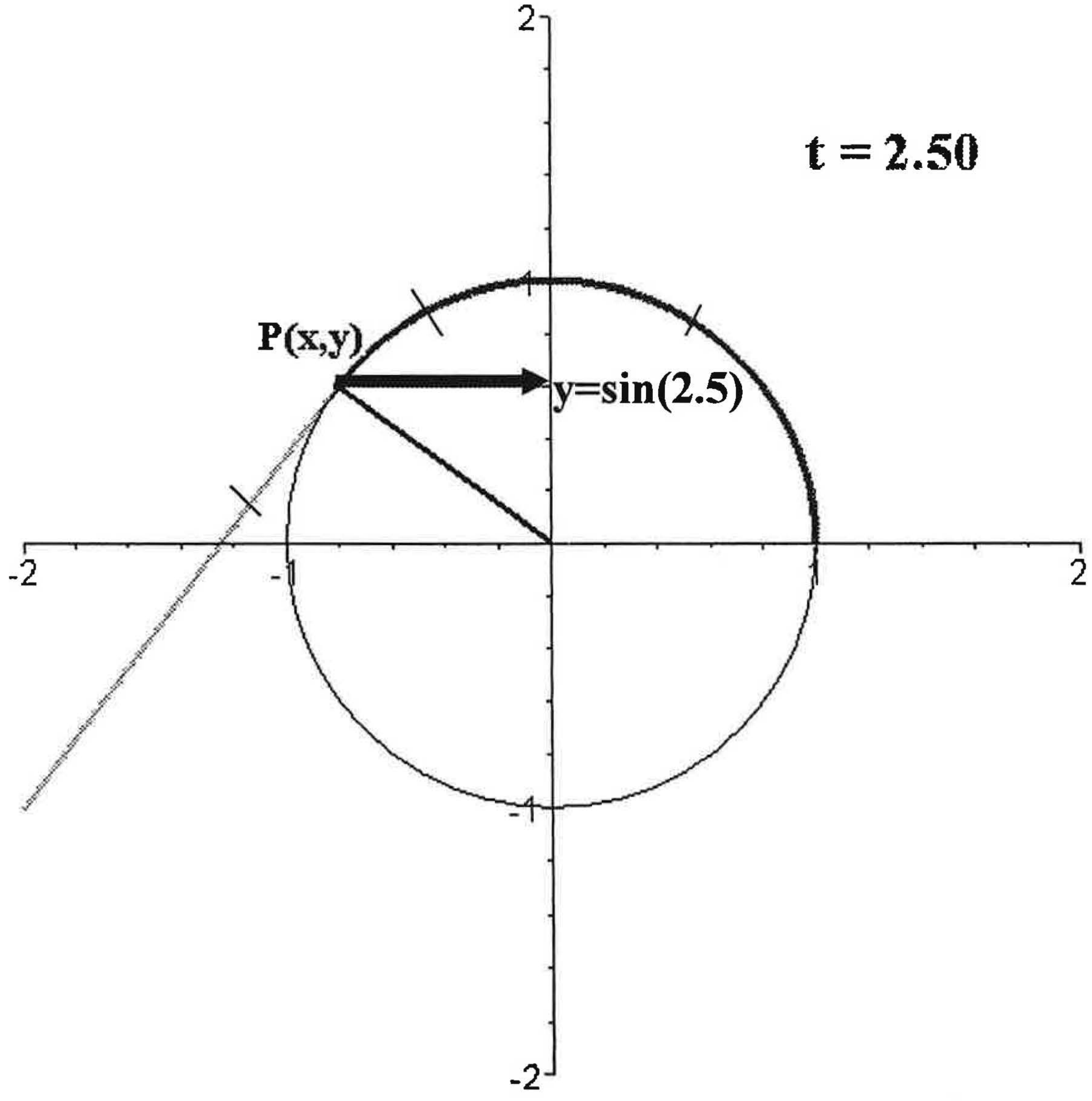
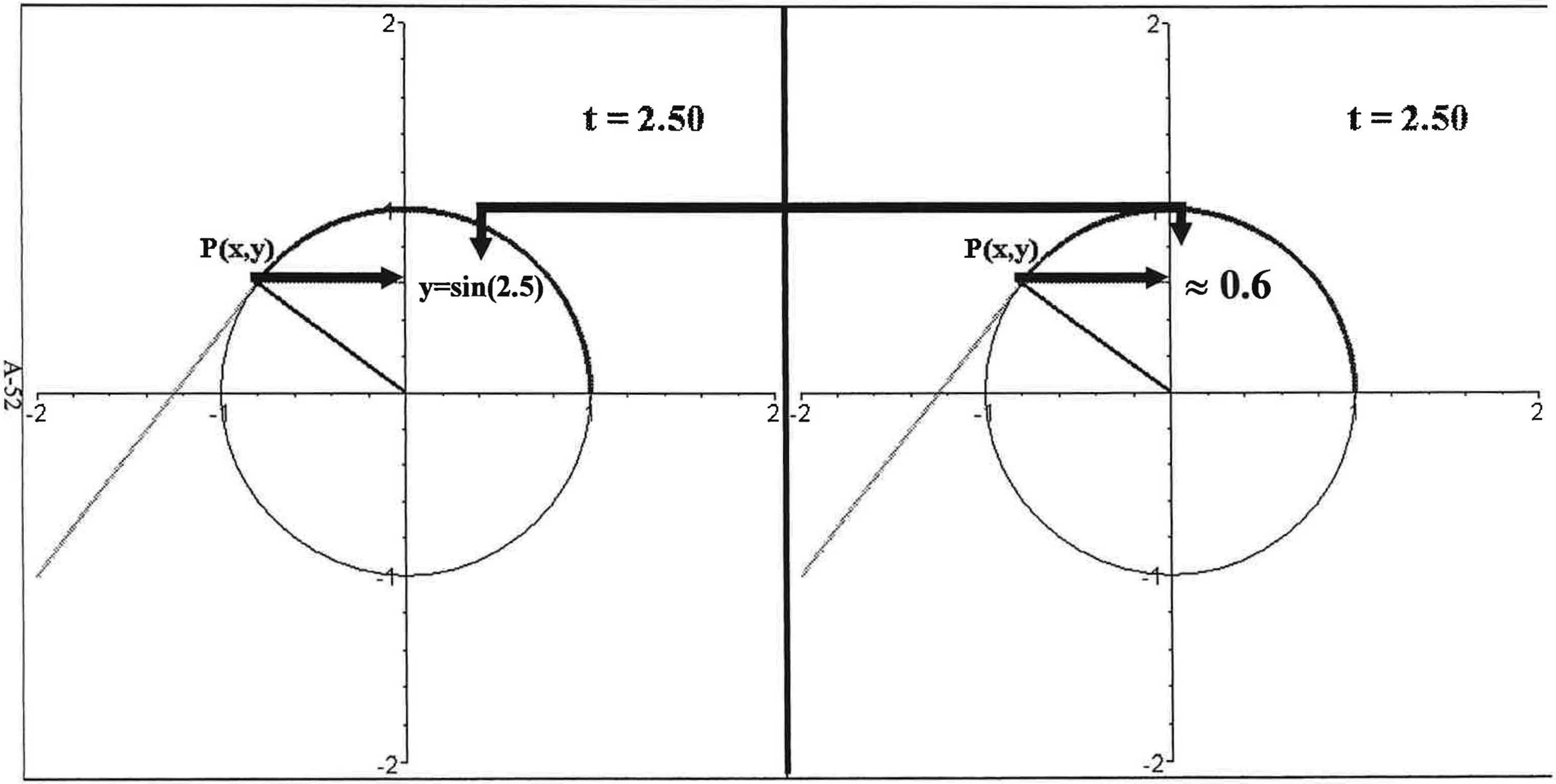
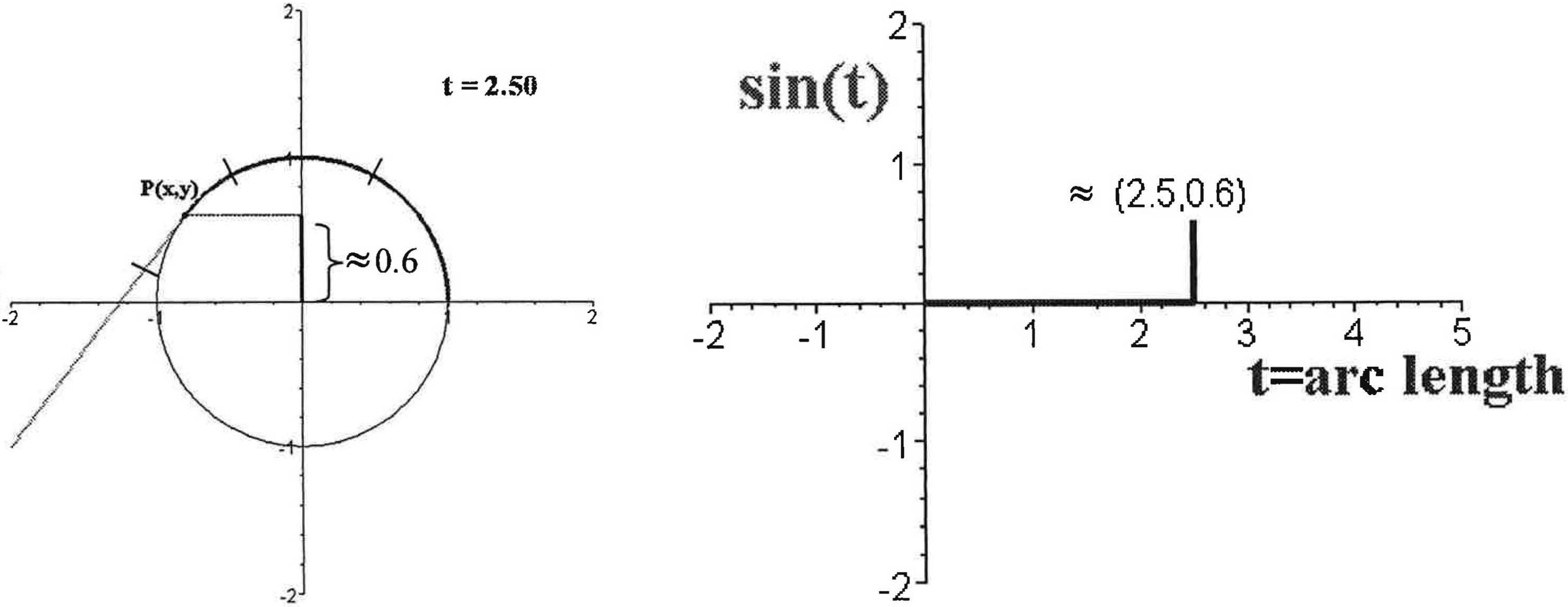


Illustration III

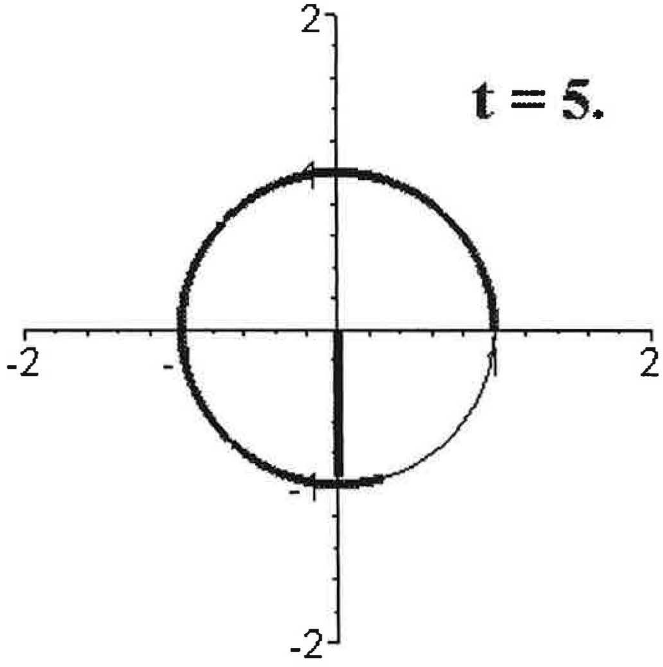


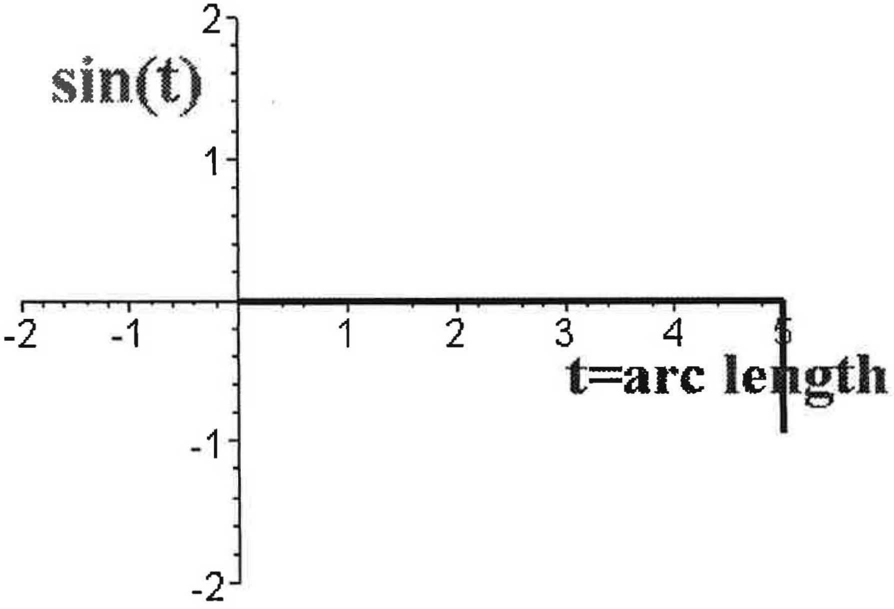
So sin(2.5) is approximately 0.6

The graph of f(t)=sin(t) plots the y-coordinate on the unit circle as a function of arc length



A-53


t = 5.
2
1
-2
-1
2
-2


sin(t)
2
1
-1
-2
-2
-1
1
2
3
4
5
t=arc length

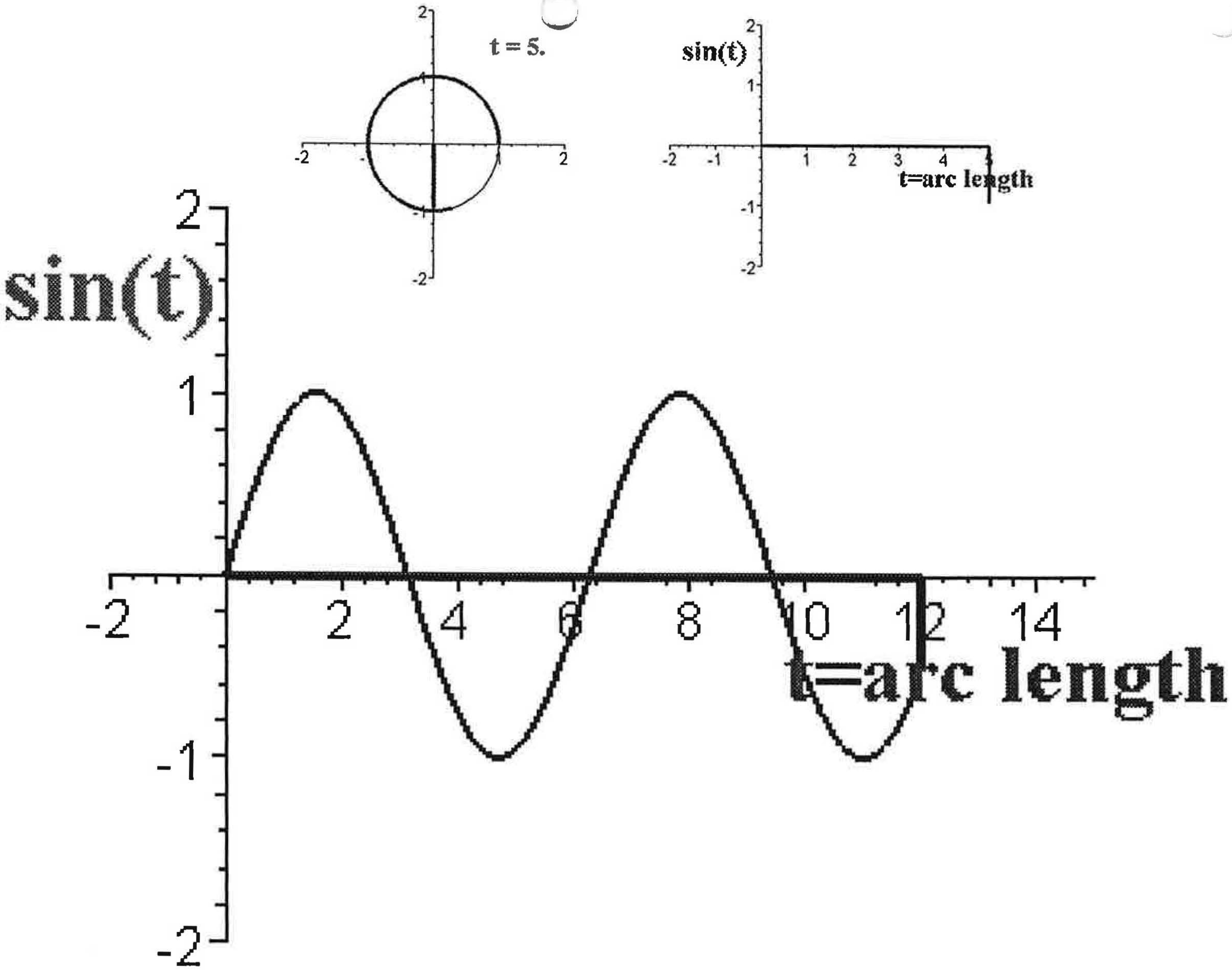
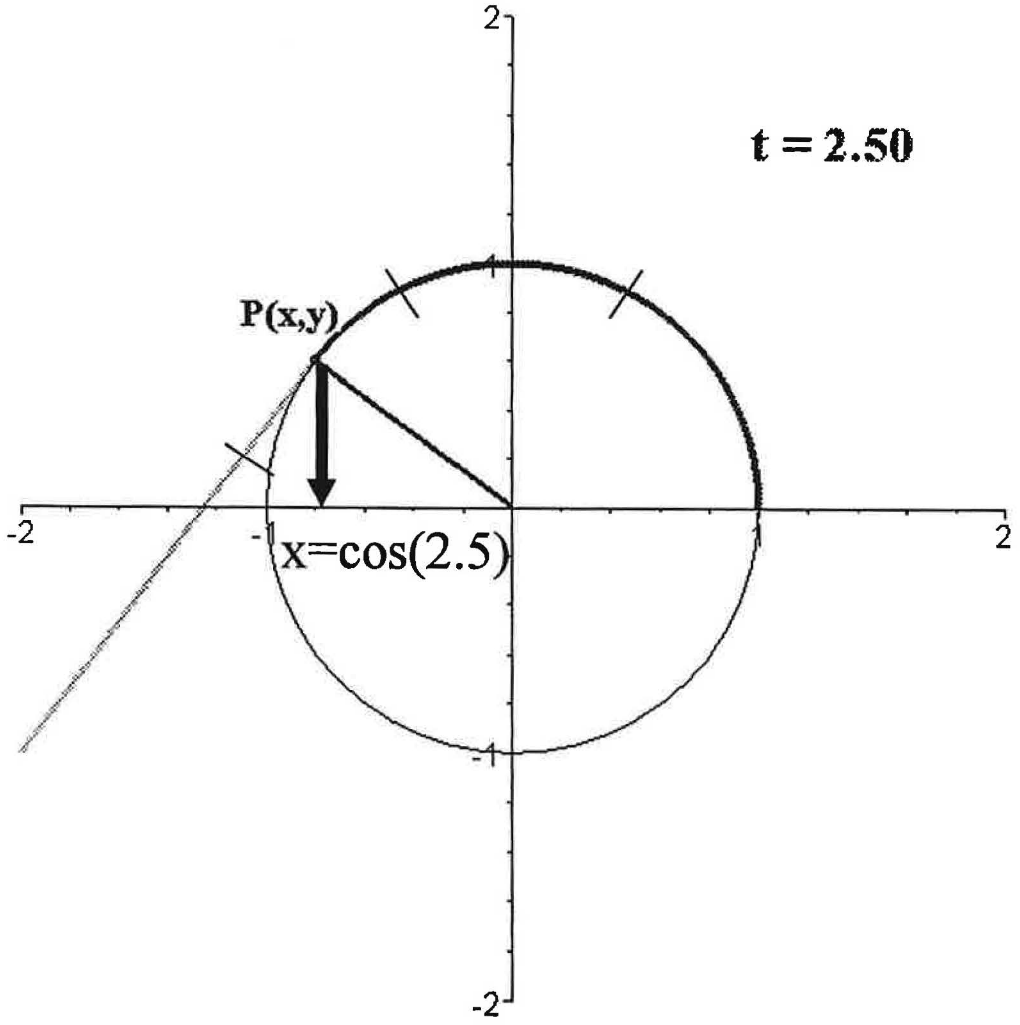
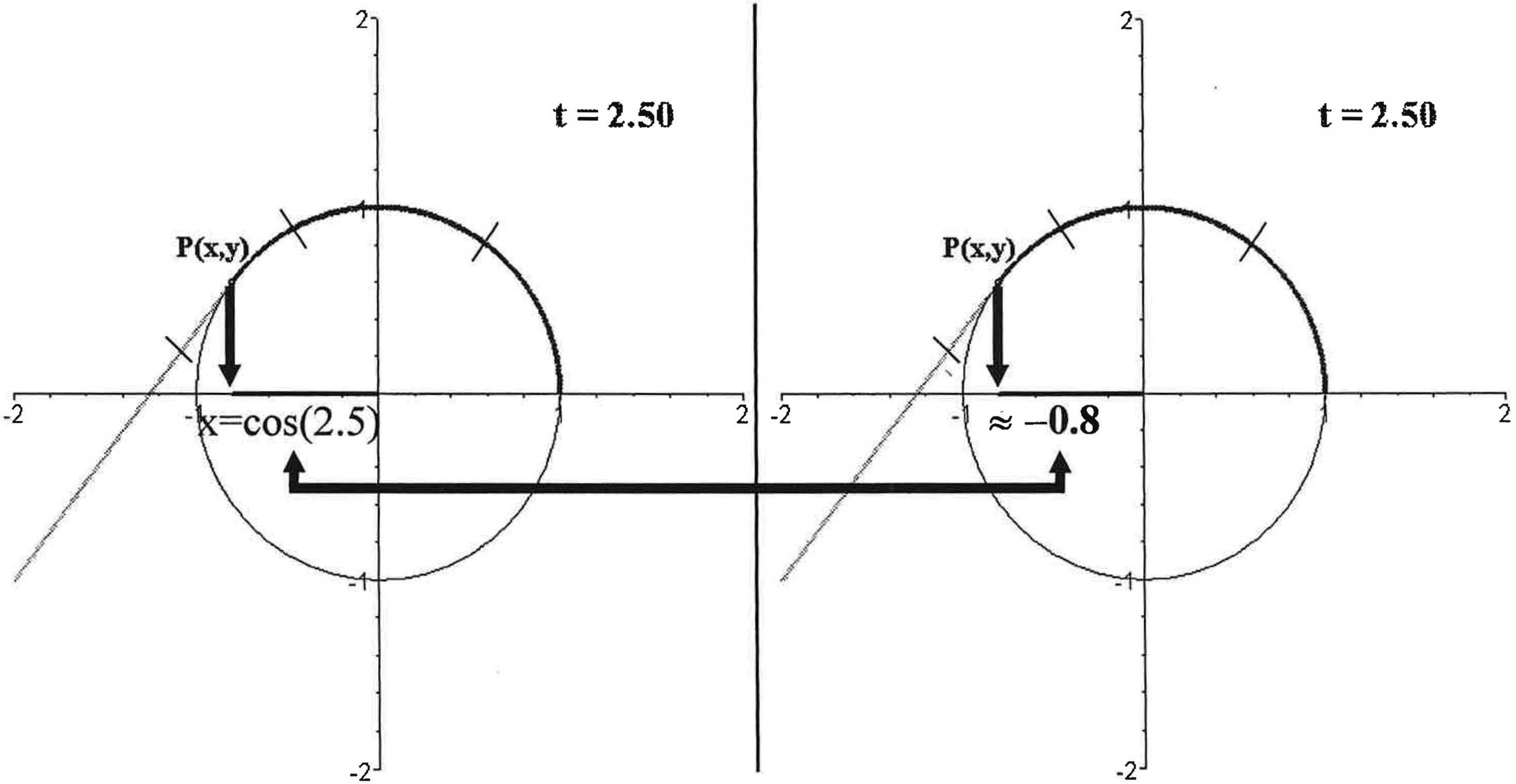


Illustration III

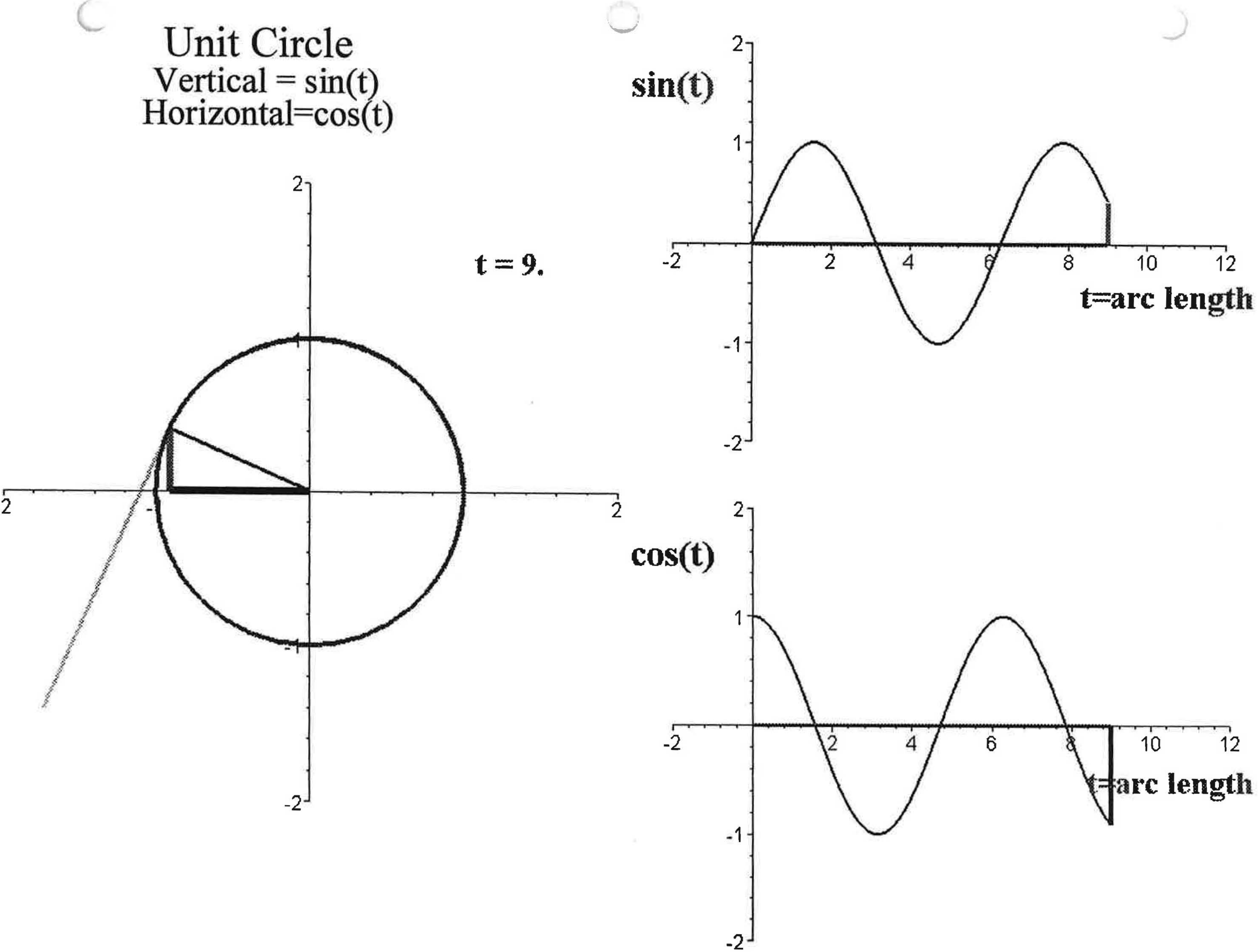
The x-coordinate of P(x,y) is the cosine of t

cos(t)=x





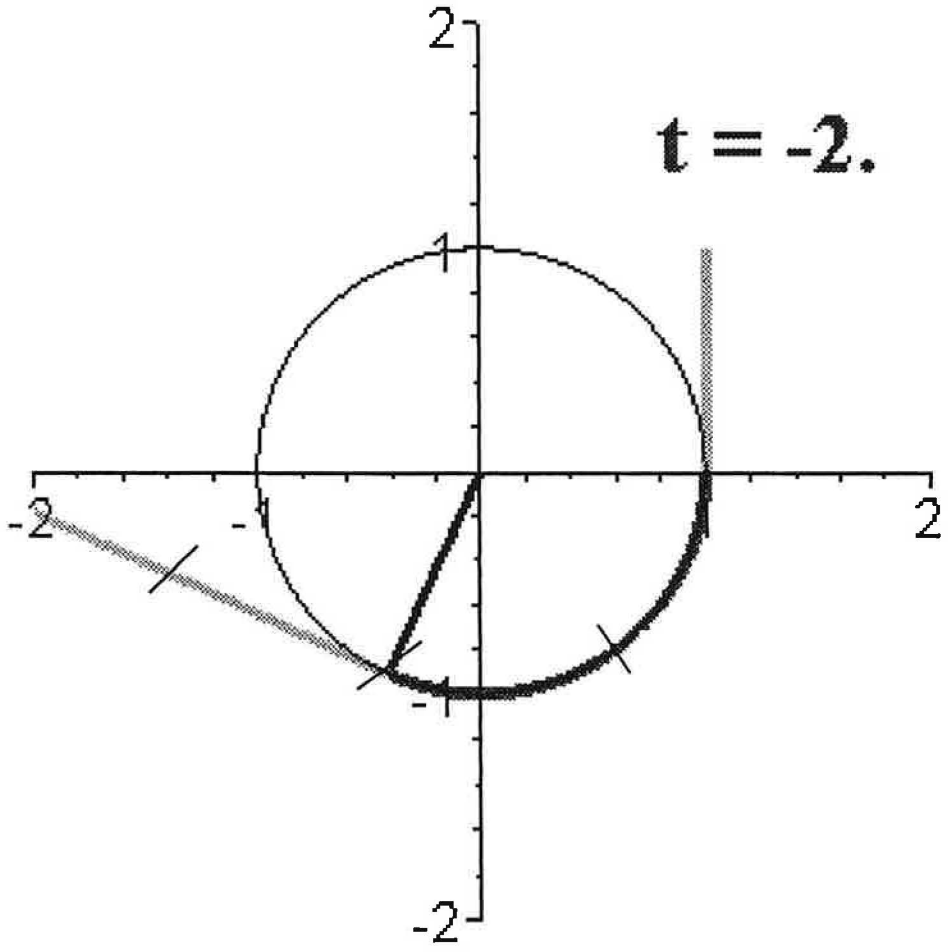
So cos(2.5) is approximately -0.8



Unit Circle
Vertical = sin(t)
Horizontal=cos(t)

Illustration III 15

Approximate sin(-2) and cos(-2)



THE END

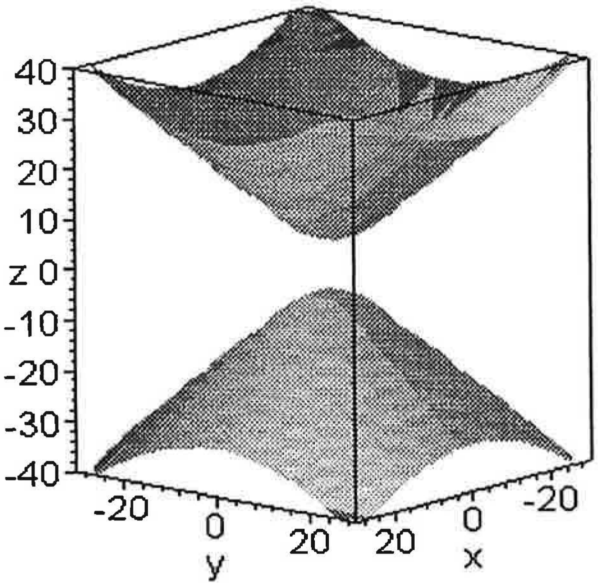
APPENDIX IV

Several Variables

A Visual Overview

Classroom Presentations

Several Variables
A Visual Overview



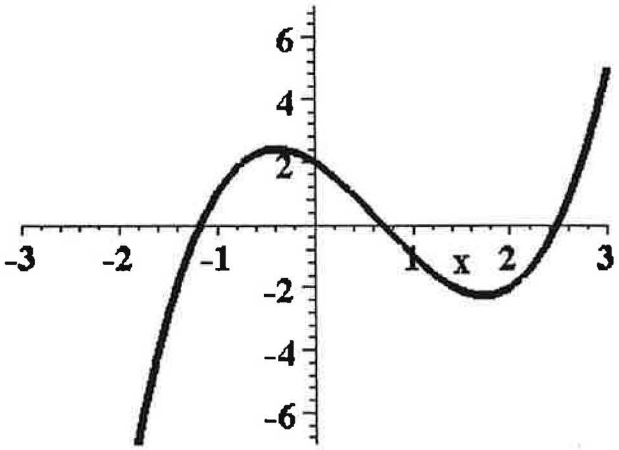
MARY CHABOT
PROFESSOR, MATHEMATICS
MT. SAN ANTONIO COLLEGE
WALNUT, CA

Introduction to several variables

Calculus of one variable

- **Graphs of functions**

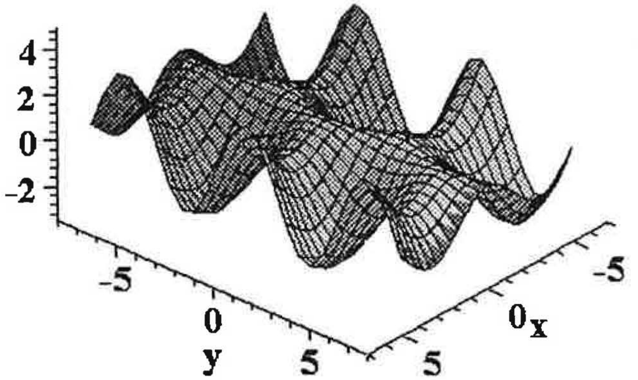
$$f(x) = x^3 - 2x^2 - 2x + 2$$



Calculus of two variables

- **Graphs of functions**

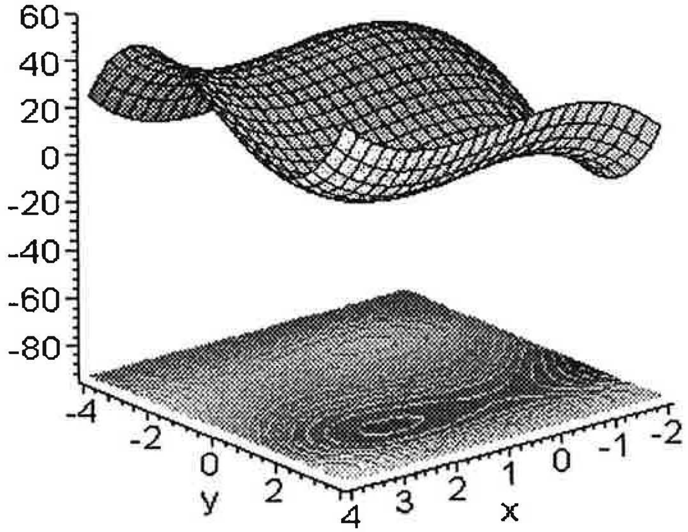
$$f(x, y) = \cos x + 0.5x \cos y$$

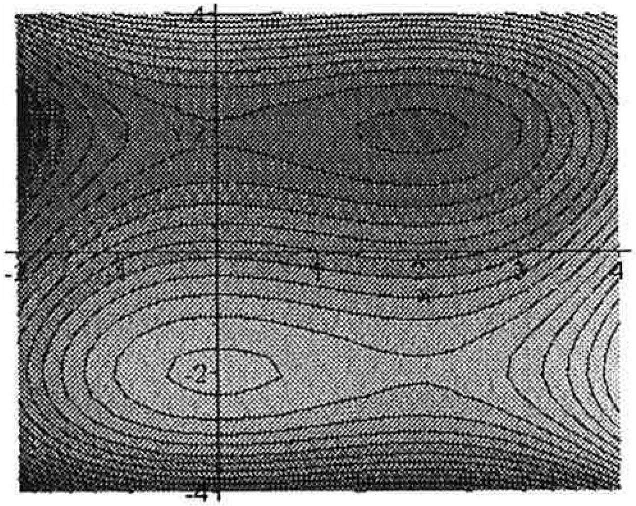


Introduction to several variables

LEVEL CURVES

(CONTOUR MAPS)





Introduction to several variables

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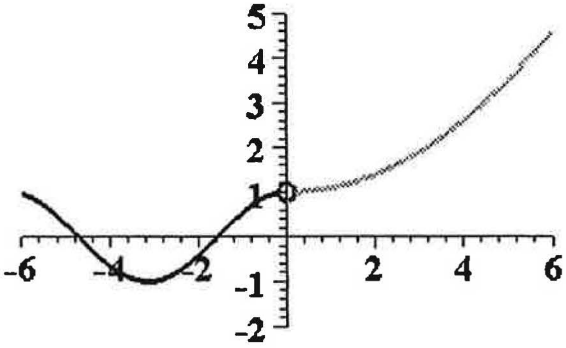
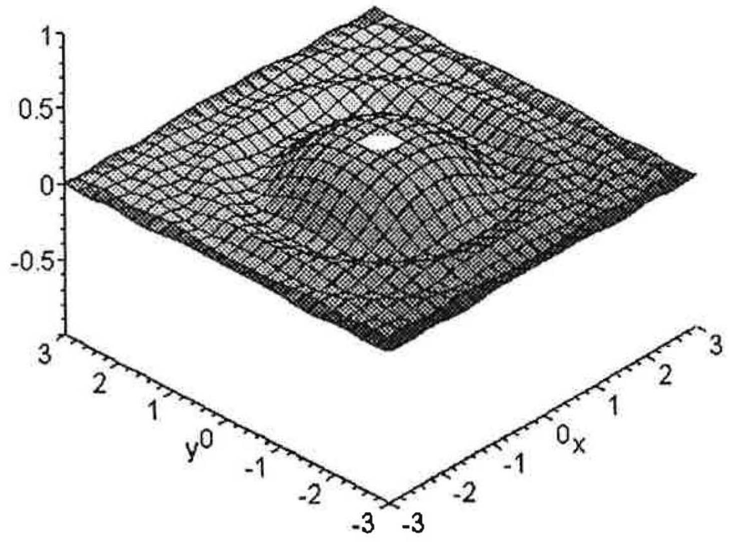
Calculus of one variable

- **Limits**



Calculus of two variables

- **Limits**



Introduction to several variables

Calculus of one variable

- **Derivative**

$$y = f(x)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Calculus of two variables

- **Derivative and Gradient**

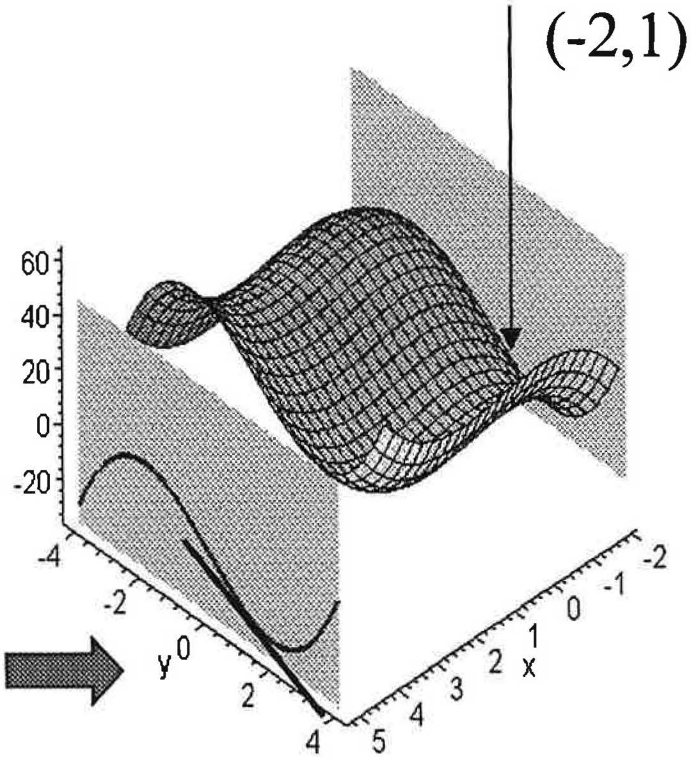
$$z = f(x, y)$$

$$f_x = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

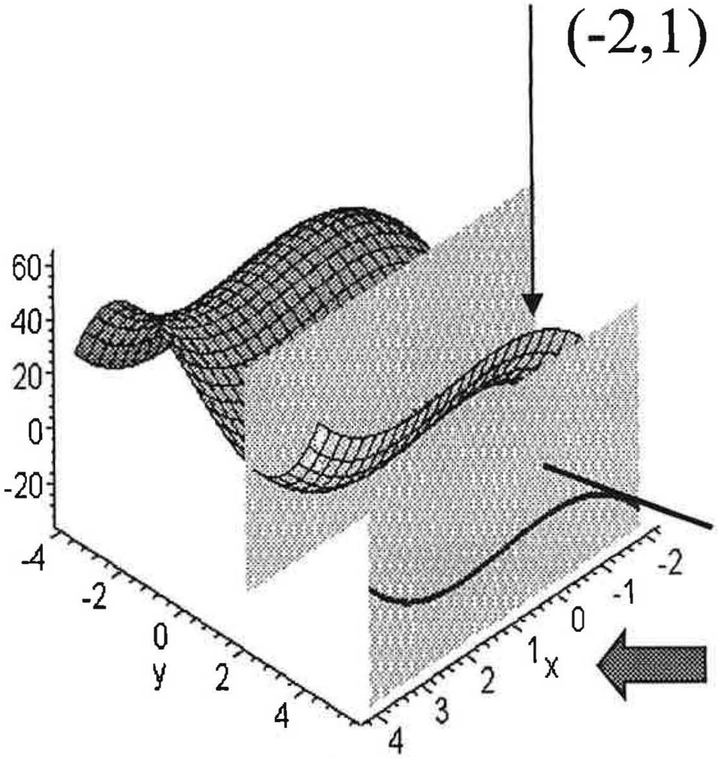
$$f_y = \lim_{k \to 0} \frac{f(x, y+k) - f(x, y)}{k}$$

$$\nabla f(x, y) = f_x \vec{i} + f_y \vec{j}$$

Introduction to several variables



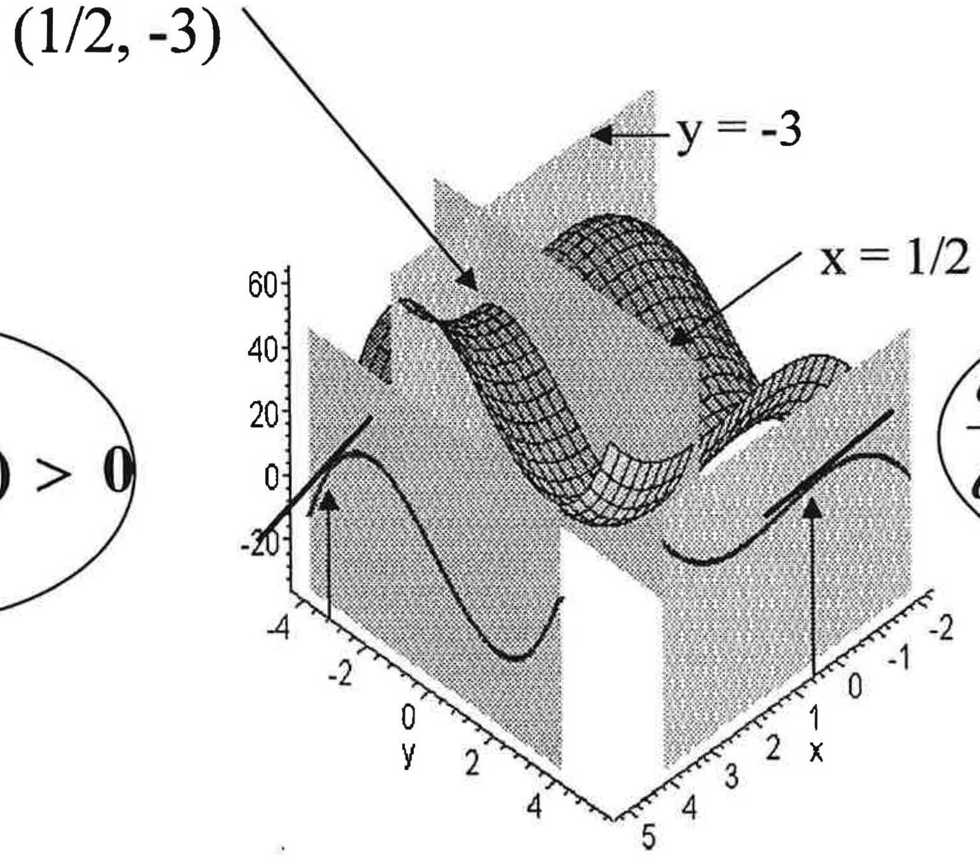
$$m_{\tan} = \boxed{\frac{\partial z}{\partial y}}(-2,1)$$



$$m_{\tan} = \boxed{\frac{\partial z}{\partial x}}(-2,1)$$

Illustration IV

Introduction to several variables



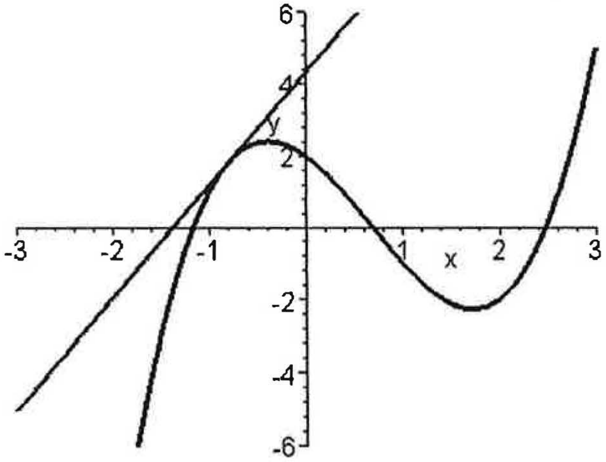
$$\frac{\partial z}{\partial y}\left(\frac{1}{2},-3\right) > 0$$

$$\frac{\partial z}{\partial x}\left(\frac{1}{2},-3\right) < 0$$

Introduction to several variables

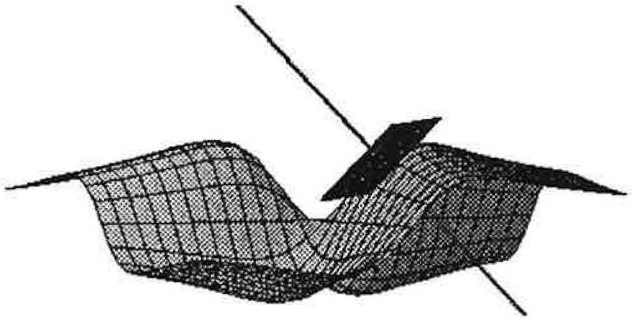
Calculus of one variable

- **Tangent Lines**



Calculus of two variables

- **Tangent Planes and normal lines**



Introduction to several variables

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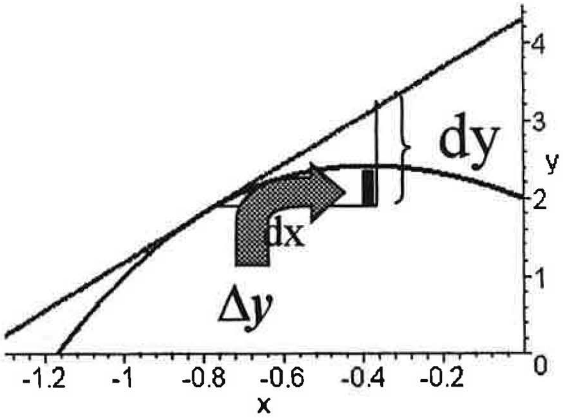
Calculus of one variable

- **Differentials**

$$y = f(x)$$

$$df = dy = \boxed{f'(x)dx}$$



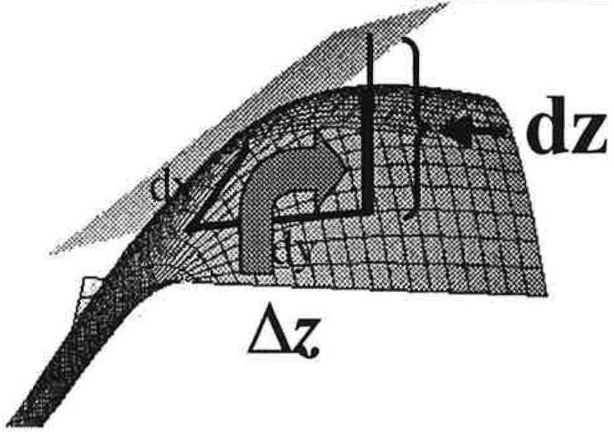
Calculus of two variables

- **Total differential**

$$z=f(x,y$$

$$df = dz = f_x dx + f_y dy$$

$$d\vec{r} = dx\vec{i} + dy\vec{j}, df = \boxed{\nabla f \bullet d\vec{r}}$$



Introduction to several variables

Calculus of one variable

- **Chain Rule**

$$y = f(x), x = x(t)$$

$$dy = f'(x)dx$$

$$dy = \frac{dy}{dx}dx$$

$$\boxed{\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}}$$

Calculus of two variables

- **Chain Rule**

$$z = f(x, y), x = x(t), y = y(t$$

$$dz = f_x dx + f_y dy$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$\boxed{\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}}$$

Introduction to several variables

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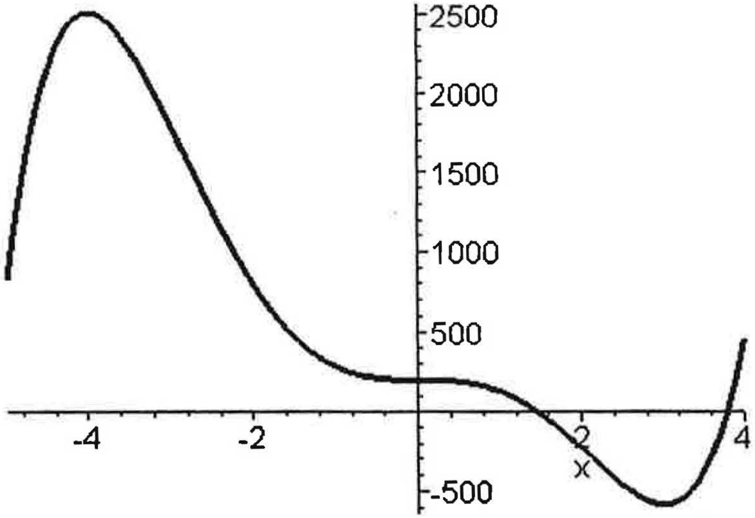
Calculus of one variable

- **Extrema**

$$f(x) = 4x^5 + 5x^4 - 80x^3 + 200$$



Calculus of two variables

- **Extrema and saddle points**

$$f(x, y) = 0.3(x^3 - 3x^2 + y^3 - 12y + 25)$$

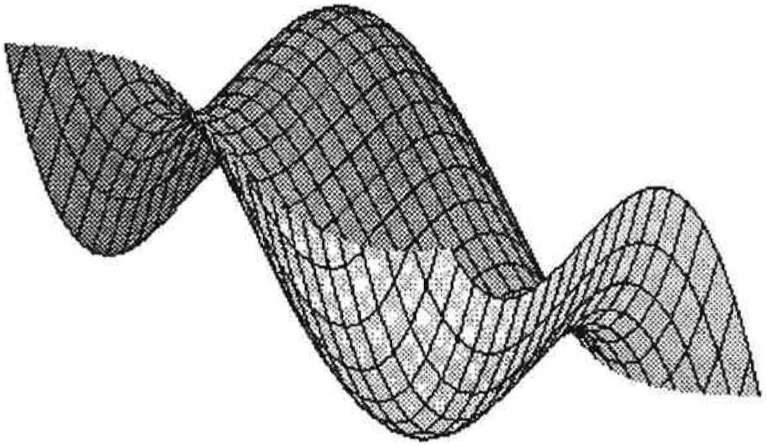
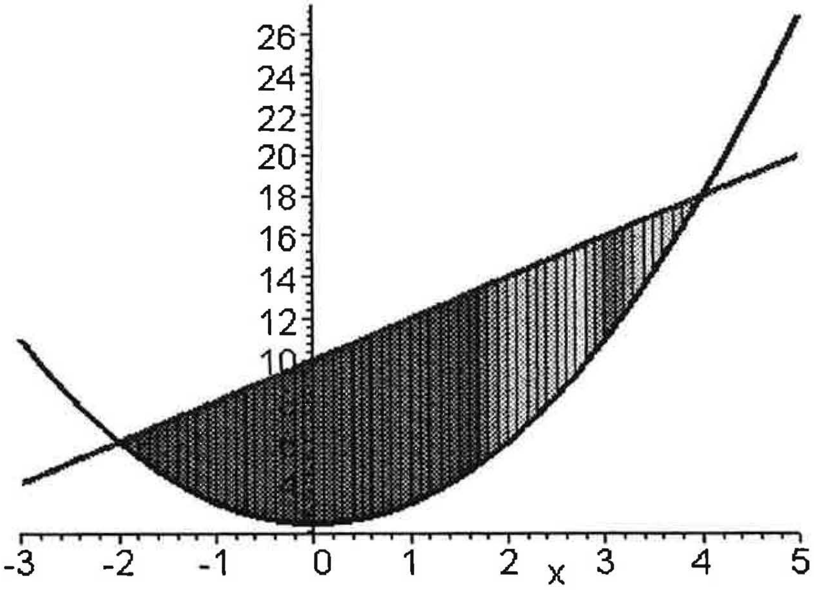


Illustration IV

Introduction to several variables

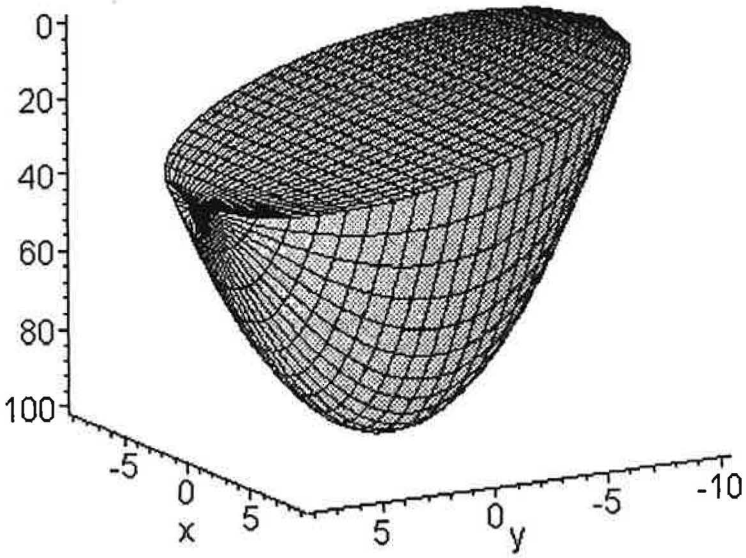
Calculus of one variable

- **Area between Curves**



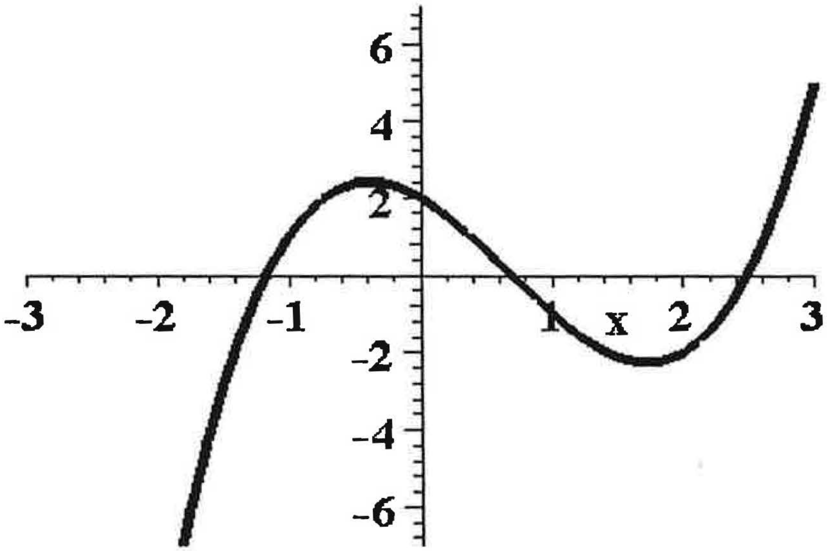
Calculus of two variables

- **Volume between surfaces**

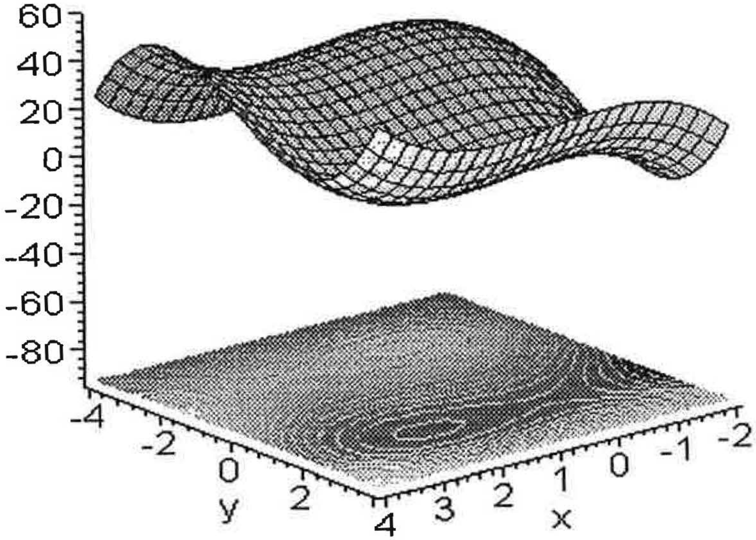


Introduction to several variables

Calculus of one variable



Calculus of two variables



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